THE
SCHOLAR'S ARITHMETIC;

OR,

FEDERAL ACCOUNTANT:

CONTAINING,

I. COMMON ARITHMETIC, the Rules and Illustrations.

II. EXAMPLES and ANSWERS with BLANK SPACES, sufficient for their operation by the Scholar.

III. To each Rule a SUPPLEMENT, comprehending, 1. QUESTIONS on the Nature of the Rule, its Use and the manner of its Operations. 2. EXERCISES.

IV. FEDERAL MONEY, with rules for all the various operations in it to reduce Federal to Old Lawful, and Old Lawful to FEDERAL MONEY.

V. Interest cast in Federal Money, with *Compound Multiplication, Compound Division*, and *Practice*, wrought in Old Lawful and in Federal Money; the same questions being put in separate columns on the same page in each kind of money, these two modes of account become contrasted, and the great advantage gained by reckoning in Federal Money easily discerned.

VI. Demonstrations by Engravings of the reason and nature of the various steps in the extraction of the Square and Cube Roots, not to be found in any other treatise on Arithmetic.

VII. Forms of Notes, Deeds, Bonds and other Instruments of Writing.

THE WHOLE IN A FORM AND METHOD ALTOGETHER NEW, FOR THE EASE OF THE MASTER AND THE GREATER PROGRESS OF THE SCHOLAR.

BY DANIEL ADAMS, M. B.

STEREOTYPE EDITION,

REVISED AND CORRECTED, WITH ADDITIONS.

KEENE, N. H.—PRINTED BY JOHN PRENTISS,

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Sold at his Bookstore, and by the principal Booksellers in the New-England States and New-York.—1825

Price 10 Dollars per dozen, 1 Dollar single.

Newhampshire District, ss.

L. S. BE IT REMEMBERED, that on the seventeenth day of July, in the thirty ninth year of the Independence of the United States of America, Daniel Adams, of *Mont Vernon,* in said District, hath deposited in this office the Title of a Book, the right whereof he claims as Author, in the following words, *to wit*:—"The *Scholar's Arithmetic:* or Federal Accountant. Containing I. Common Arithmetic, the Rules and Illustrations.—II. Examples and Answers, with blank spaces sufficient for their operation by the Scholar.—III. To each Rule, a Supplement, comprehending, 1. Questions on the nature of the rule, its use, and the manner of its operations.—2. Exercises.—IV. Federal Money, with rules for all the various operations in it, to reduce Federal to Old Lawful, and Old Lawful to Federal Money.—V. Interest cast in Federal Money with Compound Multiplication, Compound Division and Practice wrought in Old Lawful and in Federal Money, the same questions being put in separate columns on the same page in each kind of money, by which these two modes of account become contrasted, and the great advantage gained by reckoning in Federal Money easily discerned.—VI. Demonstrations by engravings of the reason and nature of the various steps in the extraction of the Square and Cube Roots, not to be found in any other treatise on Arithmetic.—VII. Forms of Notes, Deeds, Bonds, and other instruments of writing.—The whole in a form and method altogether new, for the ease of the Master and the greater progress of the Scholar.—By DANIEL ADAMS, M. B."

In conformity to the act of Congress of the United States, entitled "an act for the encouragement of Learning, by securing the copies of Maps, Charts and Books, to the authors and proprietors therein mentioned, and extending the benefit thereof to the arts of Designing, Engraving, Etching, Historical and other prints.

G. W. PRESCOTT, *Clerk of the U. S. Court, N. H. District.*

PREFACE.

IT is fourteen years since the first Edition of the Scholar's Arithmetic was offered to the Public. It has now gone through *nine* editions, and more than *Forty Thousand* copies have been circulated. In those places where it has been introduced, it never has, to the best of our knowledge, been superseded by any other work which has come in competition with it. A knowledge of these facts is, perhaps, one of the best recommendations which can be desired of the work.

It has now undergone a careful revisal. Some of the rules have been thought to be deficient in examples; in this revised edition, more than *sixty* new examples have been added under the different rules. Some have expressed a desire that answers might be given to the "*Miscellaneous Questions*," at the end of the book; these have been added accordingly, and the number of these questions increased. But what more particularly claims attention in this revised edition, is the introduction of the rule of *Exchange*, where the pupil is made acquainted with the different currencies of the several states, (that of S. Carolina and Georgia, only excepted,) and how to change these currencies from one to another; also, to Federal Money, and Federal Money to these several currencies. This has been done more particularly with a view to the accommodation of the State of New-York, and other more southern states, where this work has already acquired a very considerable circulation. Answers are given to many of the questions in different currencies, so that the pupil in N. England, N. York, &c. will find an answer to the question, each in the currency of his own particular state.

These comprehend the only additions in the present new edition.

We have now the testimony of many respectable Teachers to believe, that this work, where it has been introduced into Schools, has proved a very kind assistant towards a more speedy and thorough improvement of

Scholars in Numbers, and at the same time, has relieved masters of a heavy burden of writing out Rules and Questions, under which they have so long labored, to the manifest neglect of other parts of their Schools.

To answer the several intentions of this work, it will be necessary that it should be put into the hands of every Arithmetician: the blank after each example is designed for the operation by the scholar, which being first wrought upon a slate, or waste paper, he may afterwards transcribe into his book.

The SUPPLEMENTS to the Rules in this work are something new; experience has shown them to be very useful, particularly those "*Questions*," unanswered, at the beginning of each Supplement. These questions the pupil should be made to study and reflect upon, till he can of himself devise the proper answer. They should be put to him not only once, but again, and again, till the answers shall become as familiar with him as the numbers in his multiplication Table. The Exercises in each supplement may be omitted the first time going through the book, if thought proper, and taken up afterwards as a kind of review.

Through the whole it has been my greatest care to make myself intelligible to the scholar; such rules and remarks as have been compiled from other authors are included in quotations; the *Examples*, many of them are extracted; this I have not hesitated to do, when I found them suited to my purpose.

Demonstrations of the reason and nature of the operations in the extraction of the Square and Cube Roots have never been attempted in any work of the kind before to my knowledge; it is a pleasure to find these have proved so highly satisfactory.

Grateful for the patronage this work has already received, it remains only to be observed that no pains nor exertions shall be spared to merit its continuance.

DANIEL ADAMS.

Mont-Vernon, (N. H.) December 26th, 1815.

RECOMMENDATIONS.

New-Salem, Sept. 14th, 1801.

HAVING attentively examined "*The Scholar's Arithmetic,*" I cheerfully give it as my opinion that it is well calculated for the instruction of youth, and that it will abridge much of the time now necessary to be spent in the communication and attainment of such Arithmetical knowledge as is proper for the discharge of business.

WARREN PIERCE.
Preceptor of New-Salem Academy.

Groton Academy, Sept. 2, 1801.

SIR.....I have perused with attention "*The Scholar's Arithmetic,*" which you transmitted to me some time since. It is in my opinion, better calculated to lead students in our Schools and Academies into a complete knowledge of all that is useful in that branch of literature, than any other work of the kind I have seen. With great sincerity I wish you success in your exertions for the promotion of useful learning: and I am confident that to be generally approved your work needs only to be generally known.

WILLIAM M. RICHARDSON,
Preceptor of the Academy.

Extract of a Letter from the Hon. JOHN WHEELOCK, LL. D. *President of Dartmouth College, to the Author.*

"The Scholar's Arithmetic is an improvement on former productions of the same nature. Its distinctive order and supplement will help the learner in his progress; the part on Federal Money makes it more useful; and I have no doubt but the whole will be a new fund of profit in our country."

September 7th, 1807.

The Scholar's Arithmetic contains most of the important Rules of the Art, and something, also, of the curious and entertaining kind.

The subjects are handled in a simple and concise manner.

While the questions are few, they exhibit a considerable variety. While they are generally easy, some of them afford scope for the exercise of the Scholar's judgment.

It is a good quality of the Book, that it has so much to do with Federal Money.

The plan of showing the reasons of the operations in the extraction of the Square and Cube Roots is good.

DANIEL HARDY, JUN.
Preceptor of Chesterfield Academy.

Extract of a Letter from the Rev. LABAN AINSWORTH *of Jaffrey, to the publisher of the fourth Edition, dated August 3, 1807.*

"The superiority of the Scholar's Arithmetic to any book of the kind in my knowledge, clearly appears from its good effect in the schools I annually visit.—Previous to its introduction, Arithmetic was learned and performed mechanically; since, scholars are able to give a rational account of the several operations in Arithmetic, which is the best proof of their having learned to good purpose."

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THE

SCHOLAR'S ARITHMETIC.

INTRODUCTION.

ARITHMETIC is the art or science which treats of numbers.

It is of two kinds, *theoretical* and *practical*.

The THEORY of Arithmetic explains the nature and quality of numbers, and demonstrates the reason of practical operations. Considered in this sense, Arithmetic is a *Science.*

PRACTICAL ARITHMETIC shews the method of working by numbers, so as to be most useful and expeditious for business. In this sense Arithmetic is an *Art.*

DIRECTIONS TO THE SCHOLAR.

DEEPLY impress your mind with a sense of the importance of arithmetical knowledge. The great concerns of life can in no way be conducted without it. Do not, therefore, think any pains too great to be bestowed for so noble an end. Drive far from you idleness and sloth; they are great enemies to improvement. Remember that youth, like the morning, will soon be past, and that opportunities once neglected, can never be regained. First of all things, there must be implanted in your mind a fixed delight in study; make it your inclination; "*A desire accomplished is sweet to the soul.*" Be not in a hurry to get through your book too soon. Much instruction may be given in these few words, UNDERSTAND EVERY THING AS YOU GO ALONG.— Each rule is first to be committed to memory; afterwards, the examples in illustration, and every remark is to be perused with care. There is not a word inserted in this Treatise, but with a design that it should be studied by the Scholar. As much as possible, endeavour to do every thing of yourself; *one* thing found out by your own thought and reflection, will be of more real use to you, than *twenty* things told you by an Instructor. Be not overcome by little *seeming* difficulties, but rather strive to overcome *such* by patience and application; so shall your progress be easy and the object of your endeavours sure.

ON entering upon this most useful study, the first thing which the Scholar has to regard, is

NOTATION.

NOTATION is the art of expressing numbers by certain characters or figures: of which there are two methods. 1. The *Roman method*, by Letters 2. The *Arabic method*, by Figures. The latter is that of general use

In the Arabic method all numbers are expressed by these ten characters or figures.

1	2	3	4	5	6	7	8	9	0
Unit; or one	two;	three;	four;	five;	six;	seven;	eight;	nine;	cypher or nothing.

The nine first are called *significant figures*, or *digits*, each of which standing by itself or alone, invariably expresses a particular or certain number; thus, 1 signifies *one*, 2 signifies *two*, 3 signifies *three*, and so of the rest, until you come to *nine*, but for any number more than nine, it will always require two or more of those figures set together in order to express that number. This will be more particularly taught by

NUMERATION.

Numeration teaches how to *read* or *write* any sum or number by figures.

In setting down numbers for arithmetical operations, especially with beginners, it is usual to begin at the *right hand*, and proceed towards the *left*.

Example. If you wish to write the sum or number 537, begin by setting down the *seven*, or right hand figure, thus 7, next set down the *three*, at the left hand of the seven, thus 37, and lastly the *five*, at the left hand of the three, thus 537 which is the number proposed to be written.

In this sum thus written you are next to observe that there are *three places*, meaning the situations of the three different figures, and that each of these places has an appropriated name. The *first place*, or that of the right hand figure, or the place of the 7, is called *unit's place;* the *second place*, or that of the figure standing next to the right hand figure, in this the place of the 3, is called *ten's place;* the *third place*, or next towards the left hand, or place of the 5, is called *hundred's place;* the next or *fourth place*, for we may suppose more figures to be connected, is *thousand's place;* the next to this *tens of thousand's place*, and so on to what length we please, there being particular names for each place. Now every figure signifies differently, accordingly as it may happen to occupy one or the other of these places.

The value of the first or right hand figure, or of the figure standing in the place of *units*, in any sum or number, is just what the figure expresses standing alone or by itself; but every other figure in the sum or number, or those to the left hand of the first figure, have a different signification from their true or natural meaning; for the next figure from the right hand towards the left, or that figure in the place of *tens*, expresses so many times ten, as the same figure signifies units or ones when standing alone, that is, it is *ten times* its simple primitive value; and so on, every removal from the right hand figure, making the figure thus removed *ten times* the value of the same figure when standing in the place immediately preceding it.

Hund. Tens. Units.

Example. Take the sum 3 3 3, made by the same figure three times repeated. The first or right hand figure, or the figure in the place of *units*, has its natural meaning or the same meaning as if standing alone, and signifies *three* units or ones; but the same figure again towards the left hand in the second place, or place of tens, signifies not three units, but *three tens*, that is *thirty*, its value being increased in a *tenfold proportion;* proceeding on still further towards the left hand, the next figure or that in the third place, or place of *hundreds* signifies neither *three* nor *thirty*, but *three hundred*, which is ten times the value of that figure, in the place immediately preceding it, or that in the place of *tens*. So you might proceed and add the figure 3, fifty or

an hundred times, and every time the figure was added, it would signify ten times more than it did the last time.

A CYPHER standing alone is no signification, yet placed at the right hand of another figure it increases the value of that figure in the same ten-fold proportion, as if it had been preceded by any other figure. Thus 3, standing alone, signifies *three;* place a cypher before (30) and it no longer signifies *three*, but *thirty;* and another cypher (300) and it signifies three hundred.

The value of figures in conjunction, and how to read any sum or number agreeably to the foregoing observations, may be fully understood by the following

TABLE.

Billions.	Hund. of Thous. of Mill.	Tens of Thous. of Mill.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
2	1	6	7	2	3	5	4	2	1	8	3	6
	3	4	0	7	6	2	1	4	6	3	1	2
		1	3	0	2	5	0	3	7	6	4	5
			4	1	3	9	8	2	1	0	6	4
				2	7	0	2	1	3	6	7	5
					4	6	3	2	7	2	9	1
						1	2	3	4	6	3	2
							2	3	4	5	6	7
								8	9	0	9	8
									7	6	5	4
										1	2	3
											4	5
												7

THE words at the head of the Table shew the signification of the figures against which they stand; and the figures shew how many of that signification are meant. Thus *Units* in the first place signify *ones*, and 6 standing against it, shews that *six ones* or individuals are here meant; *tens* in the second place shew that every figure in this place means so many *tens*, and 3 standing against it, shews that *three tens* are here meant, equal to *thirty*, what the figure really signifies. *Hundreds* in the third place shew the meaning of figures in this place to be *Hundreds*, and 8 shews that *eight hundreds* are meant. In the same manner the value of each of the remaining figures in the table is known. Having proceeded through in this way, the sum of the first line of figures or those immediately against the words, will be found to be *Two Billions, one hundred sixty seven thousands, two hundred and thirty-five Millions; four hundred twenty-one thousands; eight hundred and thirty-six.* In the like manner may be read all the remaining numbers in the Table.

Those words at the head of the Table are applicable to any sum or number, and must be committed perfectly to memory so as to be readily applied on any occasion.

For the greater ease of reckoning, it is convenient and often practised in public offices, and by men of business, to divide any number into periods and half periods, as in the following manner:

TRILLIONS.	*Hundreds thous. of bill.*	*Ten thousand billions*	*Thousand billions*	*Hundred billions*	*Ten billions*	BILLIONS.	*Hundred thous. of mill.*	*Ten Thousand of mill.*	*Thousand millions*	*Hundred millions*	*Ten millions*	MILLIONS.	*Hundred thousands*	*Ten thousands*	*Thousands*	*Hundreds*	*Tens*	*Units*
5.	3	7	9,	6	3	4.	5	2	1,	7	6	8.	5	3	2,	4	6	7

The first six figures from the right hand are called the *unit period* next six the *million period*, after which the *trillion*, *quadrillion*, *quint* periods, &c. follow in their order.

Thus by the use of ten figures may be reckoned every thing whic be numbered; things, the multitude of which far exceeds the compre sion of man.

" It may not be amiss to illustrate by a few examples the extent of
" bers, which are frequently named without being attended to. If a
" son employed in telling money, reckon an hundred pieces in a mi
" and continue at work ten hours each day, he will take seventeen
" to reckon a million; a thousand men would take 45 years to reckon
" lion. If we suppose the whole earth to be as well peopled as Br
" and to have been so from the creation, and that the whole race of ma
" had constantly spent their time in telling from a heap consisting
" quadrillion of pieces, they would hardly have yet reckoned a thous
" part of that quantity."

After having been able to read correctly to his instructor, all the bers in the foregoing Table, the learner may proceed to write the fo ing numbers out in figures.

Two hundred and sixty-three.

Five thousand one hundred and sixty.

One hundred thousand, six hundred and four

Five million, eighteen thousand, seven hundred

Two million, six hundred and fifty thousand hundred and thirty-seven.

Seven hundred and ninety-four million, one dred and forty-nine thousand, five hu and twenty.

Three thousand, nine hundred and forty m four hundred and two thousand, eight hu and four.

Five hundred thirty six thousand, two hundr seventy two million, one hundred and thousand and six.

Four billion, six hundred thousand million, hundred thousand, two hundred and n two.

Explanation of the Characters made use of in this Wo

=	The sign of equality; as 100 cts=1 Dol. signifies that 10 are *equal* to 1 dollar.
+	Saint George's Cross, the sign of addition; as 2+4=6, th added to 4 are equal to 6.
—	The sign of subtraction; as 6—2=4; that is, 2 taken from 6 le
×	Saint Andrew's Cross, the sign of multiplication; as 4×6 that is, 4 times 6 are equal to 24.
÷ or)(	Reversed Parentheses, the sign of division; as 3)6(is, 6 divided by 3 the quotient is 2, or 6÷3=2.
. :: :	The sign of proportion; as 2 : 4 :: 8 : 16, that is, to 4 so is 8 to 16.

SECTION I.

FUNDAMENTAL RULES OF ARITHMETIC.

THESE are four, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; they may be either *simple* or *compound;* simple, when the numbers are all of one sort or denomination; compound, when the numbers are of different denominations.

THEY are called, *Principal or Fundamental Rules,* because all other rules and operations in arithmetic are nothing more than various uses and repetitions of these four rules.

The object of every arithmetical operation, is, by certain given quantities which are known, to find out others which are unknown. This cannot be done but by changes effected on the given numbers; and as the only way in which numbers can be changed is either by increasing or diminishing their quantities, and as there can be no increase or diminution of numbers but by one or the other of the above operations, it consequently follows, that *these four rules* embrace the WHOLE art of Arithmetic.

§ 1. SIMPLE ADDITION.

SIMPLE ADDITION is the putting together of two or more numbers, of the same denomination, so as to make them one whole or total number, called the *sum*, or *amount*.

RULE.

1. Place the numbers to be added one under another, with units under units, tens under tens, &c. and draw a line under the lowest number.
2. Add the right hand column, and if the sum be under *ten*, write it under the column; but if it be *ten*, or any *exact number* of tens, *write a cypher;* and if it be not an exact number of tens, write the *excess* above tens at the foot of the column, and for *every ten* the sum contains, *carry one* to the next column, and add it in the same manner as the former.
3. Proceed in like manner to add the other columns, carrying for the tens of each to the next, and set down the full sum of the last or left hand column.

PROOF.

Reckon the figures from the top downwards, and if the work be right, this amount will be equal to the first;—or, what is often practised, "cut "off the upper line of figures and find the amount of the rest; then if the "amount and upper line when added, be equal to the sum total, the work "is supposed to be right."

EXAMPLES.

	Thous.	Hund.	Tens.	Units.		Thous.	Hund.	Tens.	Units.		Hund.	Tens.	Units.		Units.
1. What will be the amount of . . .	3	6	1	2	dolls.	8	0	4	3	dolls.	6	5	1	dolls. and of	3

dollars when added together ?

Here are *four* sums given for addition; two of them contain *units*, *tens*, *hundreds*, *thousands*; another of them contains *units*, *tens*, *hundreds*; and a fourth contains *units* only. The first step to prepare these sums for the operation of addition, is to write them down, units under units, tens under tens, &c. thus ;—

	T. of Thou.	Thousands.	Hundreds.	Tens.	Units.	
		3	6	1	2	dollars
		8	0	4	3	dollars
			6	5	1	dollars
					3	dollars
Answer, or Amount,	1	2	3	0	9	dollars
Amount of the three lower lines,		8	6	9	7	
Proof,	1	2	3	0	9	

To find the answer or amount of the sums given to be added, begin with the right hand column, and say, 3 and 1 make 4, and 3 are 7, and 2 are 9, which sum (9) being less than *ten*, set down directly under the column you added. Then proceeding to the next column, say again; 5 and 4 are 9, and 1 is 10; being *even ten*, set down 0, and carry one to the next column, saying 1, which I carry to 6 is 7, and 0 is nothing, but 6 make 13; which sum (13) is an excess of 3 over even ten; therefore set down 3 and carry 1 for the ten to 8 in the next column, saying 1 to 8 is 9, and 3 are 12; this being the last column, set down the whole number (12) placing the 2, or unit figure directly under the column, and carrying the other figure, or the 1, forward to the next place on the left hand, or to that of *Tens of thousands*, and the work is done.

It may now be required to know if the work be right. To exhibit the method of proof let the upper line of figures be cut off as seen in the example. Then adding the three lower lines which remain, place the amount (8697) under the amount first obtained by the addition of all the sums, observing carefully that each figure fall directly under the column which produced it; then add this last amount to the upper line which you cut off; thus 7 to 2 are 9; 9 to 1 are 10; carry one to 6 is 7 and 6 are 13; 1 which I carry again to 8 is 9 and 3 are 12, all which being set down in their proper places, and as seen in the example, compare the amount (12309) last obtained, with the first amount (12309) and if they agree, as it is seen in this case they do, then the work is judged to be right.

Note.—The reason of *carrying for ten* in all simple numbers is evident from what has been taught in Notation. It is because 10 in an inferior column is just equal in value to 1 in a superior column. As if a man should be holding in his *right hand* half pistareens, and in his left, dollars. If you should take 10 half pistareens from his right hand, and put one dollar into his left hand, you would not rob the man of any of his money, because 1 of those pieces in his left hand is just equal in value to 10 of those in his right hand.

2

1	6	7	5	2
5	4	0	3	8
2	6	3	7	1

3

6	0	3	7
2	4	8	0
2	6	5	1

4

3	4	7	1	2	6
5	7	0	3	2	8
4	2	1	6	8	3

5

1	6	4	3	7	0	1
6	3	5	4	2	8	6
2	1	3	2	4	3	2
7	5	2	6	3	1	9

6

3	6	2	1	5	2	4	3
6	3	0	9	8	1	7	5
2	1	4	3	1	0	6	4
8	5	1	7	6	4	5	0

7

6	3	9	8	7	5	1
4	6	8	2	3	7	6
2	8	7	5	4	1	0
6	7	8	5	4	0	3
2	6	4	7	3	1	0

8

3	4	5	6	7	8	9	2
9	8	6	5	3	5	1	0
7	3	8	7	9	5	2	8
9	6	7	4	9	8	0	1
1	3	2	0	1	2	3	6

9

4	2	6	1	7	8	3
6	4	0	2	5	3	1
	3	5	2	6	4	3
		7	6	2	3	4
		1	2	5	0	7
			4	6	8	2
				1	3	5
					6	8
						7

10

5	3	8	6	5	0	7	6
		9	8	3	1	2	4
4	0	7	5	0	2	0	8
				1	6	3	4
	8	0	1	7	6	0	0
			2	9	2	0	1
		3	6	4	8	3	5
			3	6	1	0	0
						7	3

SUPPLEMENT TO ADDITION.

THE attentive scholar who has understood, and still carries in his mind, what has already been taught him of Addition, will be able to answer his instructor to the following

QUESTIONS.

1. What is simple Addition?
2. How do you place numbers to be added?
3. Where do you begin the addition?
4. What is the answer called?
5. How is the sum or amount of each column to be set down?
6. What do you observe in regard to setting down the sum of the last column?
7. Why do you carry for ten rather than any other number?
8. How is addition proved?

NOTE. Should the learner find any difficulty in giving an answer to the above questions, he is advised to turn back and consult his Rule, with its illustrations.

EXERCISES.

1. What is the amount of 2801 dollars; 765 dollars; and of 397 dollars, when added together? *Ans.* 3963 *dollars.*

2. Suppose you lend a neighbour £210 at one time, £76 at another, £17 at another, and £9 at another. What is the sum lent? *Ans.* £312.

NOTE. The scholar who looks at greatness in his class, will not be discouraged by a little difficulty which may at first occur in stating his question, but will apply himself the more closely to his Rule, and to thinking, that if possible he may be able himself to answer what another may be obliged to have taught him by his instructor.

3. A tree was broken off by the wind, 27 feet from the ground; the part broken off was 71 feet long; what was the height of that tree before it was broken? *Ans.* 98 *feet.*

4. A man being asked his age, said he was 27 years old when he married, and he had been married 15 years. What was the man's age? *Ans.* 42.

5. WASHINGTON was born in the year of our Lord 1732; he was 67 years old when he died; in what year of our Lord did he die?

Ans. 1799.

6. There are two numbers; the less number is 8761, the difference between the numbers is 597; what is the greatest number?

Ans. 9358.

7. From the Creation to the departure of the Israelites from Egypt was 2513 years; to the siege of Troy, 307 years more; to the building of Solomon's temple, 180 years; to the building of Rome, 251 years; to the expulsion of the kings from Rome, 244 years; to the destruction of Carthage, 363 years; to the death of Julius Cæsar, 102 years; to the Christian æra, 44 years; required the time from the Creation to the Christian æra?

Ans. 4004.

8. At the late Census, taken A. D. 1810, the number of inhabitants in the several *New-England States* was as follows; viz. *Maine*, 228705; *N. Hampshire*, 214460; *Vermont*, 217895; *Massachusetts*, 472040; *Rhode-Island*, 76931; *Connecticut*, 261942; what was the number of inhabitants at that time in New-England?

Ans. 1471973.

9. There are five numbers, the first is 2617; the second 893; the third 1702; the fourth as much as the three first; and the fifth twice as much as the third and fourth; what is the whole sum?

Ans. 24252.

10. A gentleman left his son, 2475 dollars more than his daughters, whose fortune was 25 thousand, 25 hundred, and 25 dollars; what was the son's portion, and what was the amount of the whole estate?

Ans. Son's portion, 30000.
Whole estate, 57525.

§ 2. SIMPLE SUBTRACTION.

SIMPLE SUBTRACTION is taking a less number from a greater of the same denomination, so as to shew the difference or remainder; as 5 taken from 8, there remains 3.

The greater number (8) is called the *Minuend*, the less number (5) the *Subtrahend*, and the difference (3) or what is left after Subtraction, the *Remainder*.

RULE.

"Place the less number under the greater, units under units, tens under tens, and so on. Draw a line below; then begin at the right hand, and subtract each figure of the less number from the figure above it, and place the remainder directly below. When the figure in the lower line exceeds the figure above it, suppose 10 to be added to the upper figure; but in this case you must add 1 to the under figure in the next column before you subtract it. This is called, *borrowing ten.*"

PROOF.

Add the remainder and subtrahend together, and if the sum of them correspond with the minuend, the work is supposed to be right.

Minuend	8	6	5	3
Subtrahend	5	2	7	1
Remainder	3	3	8	2
Proof	8	6	5	3

The numbers being placed with the larger uppermost, as the rule directs, I begin with the unit or right hand figure in the subtrahend, and say, 1 from 3 there remain 2, which I set down, and proceeding to tens, or the next figure, 7 from 5 I cannot, I therefore borrow, or suppose ten to be added to the upper figure (5) which make 15, then I say 7 from 15 and there remain 8, which I set down: then proceeding to the next place, I say, 1 which I borrowed to 2 is 3, and 3 from 6 and there remain 3; this I set down, and in the next place I say 5 from 8 and there remain 3, which I set down and the work is done.

Proof. I add the remainder to the subtrahend, and finding the sum just equal to the minuend, suppose the work to be right.

Note. The reason of *borrowing ten*, will appear if we consider, that, when two numbers are equally increased by adding the same to both, their difference will be equal. Thus the difference between 3 and 5 is 2; add the number 10 to each of these figures (3 and 5) they become 13 and 15, still the difference is 2. When we proceed as above directed, we add or suppose to be added, 10 to the *minuend*, and we likewise add one to the next higher place of the *subtrahend*, which is just equal in value to 10 of the lower place.

2.	From	3	2	7	8	6	5	3	2	1	4	6	5	the minuend,
	Take	1	0	6	7	9	3	6	1	2	3	4	2	the subtrahend.

Remainder.

Note.—In case of *borrowing ten*, it is a matter of indifference, as it respects the operation, whether we suppose ten to be added to the upper figure, and from the sum subtract the lower figure and set down the difference; or, as Mr. PIKE directs, first subtract the lower figure from 10, and adding the difference to the figure above, set down the sum of this difference and the upper figure. The latter method may perhaps be thought more easy, but it is conceived, that it does not lead the understanding of youth so directly into the nature of the operation as the former.

1. From	1 0 2 3 6 7 4 2 3 1 7 9 8 1 0 6 2
Take	8 7 9 1 2 8 4 5 0 6 7 0 3 2 8 1
Rem.	

2. From	1 0 2 3 6 7 4 2 3 1 7 9 8 1 0 6 2
Take	8 7 9 1 2 8 4 5 0 6 7 0 3 2 8 1
Rem.	

3. From 21468317012101, take 568497067382. Rem. 20899819944719

4. From 364710825193, take 279403865746. Rem. 85306959447.
5. From 168012372458, take 89674807683. Rem. 78337564775.
6. From 100610528734, take 99874197867. Rem. 736330867.
7. From 628103570126, take 248976539782. Rem. 379127030344.
8. From 10000, take 9999. Rem. 1. 9. From 10000, take 1. Rem. 9999.

The distance of time since any remarkable event, may be found by subtracting the date thereof from the present year.

EXERCISES.

1. How long since the American Independence, which was declared in 1776?

	1	8	1	7 present time.
	1	7	7	6 date of Ind.
Ans.			4	1 *years since.*

2. King Charles, the martyr, was beheaded 1648: how many years is it since?

So, likewise, the distance of time from the occurrence of one thing to that of another, may be found by subtracting the date of the thing first happening, from that of the last.

EXAMPLE.

1. How long from the discovery of America by Columbus, 1492, to the commencement of the war, 1775, which gained our Independence?

	1	7	7	5
	1	4	9	2
Ans.		2	8	3 *years.*

2. How long from the termination of the war in 1783, which gained our Independence, to the commencement of the last war between the United States and Great Britain in 1812? *Ans.* 29 *years.*

SUPPLEMENT TO SUBTRACTION.

QUESTIONS.

1. What is Simple Subtraction?
2. How many numbers must there be given to perform that operation?
3. How must the given numbers be placed?
4. What are they called?
5. When the figure in the lower number is greater than that of the upper number from which it is to be taken, what is to be done?
6. How does it appear that in subtracting a less number from a greater, the occasional *borrowing of ten* does not affect the difference between these two numbers?
7. How is subtraction proved?

EXERCISES.

1. What is the difference between 78360 and 5421? *Ans.* 72939.

2. From a piece of cloth that measured 691 yards, there were sold 273 yards; how many yards should there remain? *Ans.* 418.

3. There are two numbers, whose difference is 375; the greater number is 862; I demand the less? *Ans.* 487.

4. What number is that which taken from 175 leaves 96? *Ans.* 79.

5. Suppose a man to have been born in the year 1745, how old was he in 1799? *Ans.* 54 *years.*

6. What number is that to which if you add 789 it will become 6350? *Ans.* 5561.

7. Supposing a man to have been 63 years old in the year 1801; in what year was he born? *Ans. in the year* 1738.

8. At the census in 1800, the number of inhabitants in the New-England States was 1233011; at the late census in 1810, the number was 1471937; What was the increase of the population in the New-England States in the ten years between 1800 and 1810? *Ans.* 238926.

§ 3. SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION teaches, having two numbers given of the same denomination, to find a third which shall contain either of the two given numbers as many times as the other contains a unit. Thus, 8 multiplied by 5, or 5 times 8 is 40—The given numbers (8 and 5) spoken of together, are called *Factors*. Spoken of separately, the first or largest number (8) or number to be multiplied, is called the *Multiplicand;* the less number (5) or number to multiply by, is called the multiplier, and the amount (40) the *Product*.

Before any progress can be made in this rule, the following Table must be committed perfectly to memory.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

By this table the product of any two figures will be found in that square which is on a line with the one and directly under the other. Thus, 56 the product of 7 and 8, will be found on a line with 7 and under 8: so 2 times 2 is 4; 3 times 3 is 9, &c.—In this way the table must be learned and remembered.

RULE.

1. Place the numbers as in Subtraction, the larger number uppermost with units under units, &c. and then draw a line below.

2. *When the multiplier does not exceed* 12: begin at the right hand of the multiplicand, and multiply each figure contained in it by the multiplier, setting down all over *even tens* and carrying as in addition.

3. *When the multiplier exceeds* 12; multiply by each figure separately, first by the *units* of the multiplier, as directed above, then by the *tens*, and the other figures in their order, remembering always to place the first figure of each product directly under the figure by which you multiply; having gone through in this manner with each figure in the multiplier, add their several products together, and the sum of them will be the product required.

EXAMPLES.

1. Multiply 5291 by 3.

OPERATION.

5 2 9 1	Multiplicand.
3	Multiplier.
1 5 8 7 3	Product.

The numbers being placed as seen under the operation, say—3 times 1 is 3; which set down directly under the multiplier; then 3 times 9 is 27; set down 7 and carry 2. Again 3 times 2 is 6 and 2 I carry is 8; set down 8: then lastly, 3 times 5 is 15, which set down, and the work is done.

2. What is the product of 4175 multiplied by 37?

Place the Factors thus,	4 1 7 5	Multiplicand.
	3 7	Multiplier.
	2 9 2 2 5	Prod. by the *unit* (7) of the multiplier
	1 2 5 2 5	Product by the *tens* (3)
	1 5 4 4 7 5	Product or answer.

In this example, as the multiplier exceeds 12, therefore you must multiply by each figure separately. First, by the units (7) just in the manner of the other example. Secondly by the tens (3) in the same way excepting only, that the first figure of the product in the multiplication by 3, must be placed under the 3, that is, under the figure by which you multiply.

Lastly, add these two products together, and the sum of them is the answer.

Proof.—The better way of proving Multiplication is by Division: but till the pupil shall have been instructed in that rule he may make use of the following

METHOD.

Cast the *nines* out of the *Multiplicand*, and set the remainder at the right hand of a cross; do the same with the *Multiplier* and set the remainder at the left hand of the cross; then multiply the figures at the right and left of the cross together, cast the nines out of the product, and set the remainder over the cross; also cast the nines out of the answer or product of the multiplicand and multiplier, and set the remainder under the cross, which will be the same as that over it if the work be right.

Note.—*To cast the nines out of any number*, proceed thus: beginning at the right hand of the number, add the figures; when the sum exceeds 9, drop the sum and begin anew, by adding, first, the figures which would express it. Pass by the nines, and when the sum comes out exactly 9, neglect it; what remains after the last addition, will be the remainder sought; for example—suppose it be required to cast the 9's out of 576394, proceed thus:—5 to 7 is 12, which sum (*twelve*) as it exceeds 9 you must drop, and beginning anew, first add the figures (12) which would express *twelve*, saying 1 to 2 is 3 (proceeding with the other figures which remain to be added) and 6 are 9, being *exactly nine*, neglect it, and begin again; 3 to 9 are *twelve;* again drop the sum (*twelve*) and add the figures (12) which would express it, 1 to 2 is 3 and 4 are 7, which sum (7) is the remainder after the last addition, or the thing sought, and is the remainder that would be left after dividing the sum 576394 by 9.

3. Multiply 7 6 5 3 0 2 Multiplicand.
6 5 Multiplier.

3 8 2 6 5 1 0
4 5 9 1 8 1 2

4 9 7 4 4 6 3 0 Product.

PROOF.

1
2 × 5
1

Proof. I cast the 9's out of the Multiplicand and set the remainder (5) at the right hand of the cross; I do the same with the Multiplier, and set the remainder (2) at the left hand of the cross; these remainders I multiply together, and casting out the 9's from the product (10) the remainder is 1, which I set at the top of the cross; I then cast out the 9's from the Product (49744630) and set the remainder (1) at the bottom of the cross, which as it agrees with the remainder at top, I suppose the work to be right.

There is nothing more easy than proving Multiplication by this method so soon as the scholar shall have given it such attention, as to make it a little familiar.

Note. Should the Multiplier or Multiplicand, either or both, be less than 9, they are to be taken as the remainders.

4. Multiply 37846 by 235. Product, 8893810.

5. Multiply 14356 by 648 Product, 9302688.

6. Multiply 29831 by 952. Product, 28399112.

7. Multiply 93956 by 8704. Product, 817793024.

8.	Multiply 9 8 7 0 4 5 6 3	by 7 5 6 0 4	Prod.	7462459781052
9.	- 3 4 6 2 3 2 1	- 9 6 4 8 4	-	334058579364
10.	- 5 2 7 5 3 5	- 1 5 7 2 8	-	8297070480
11.	- 2 7 5 8 2 7	- 1 9 7 2 5	-	5440687575
12.	- 6 9 6 3 7 4	- 4 6 3 9 5 7	-	323087591918

Contractions and varieties in Multiplication.

Any number which may be produced by the multiplication of two or more numbers, is called a *composite number.* Thus 15 which arises from the multiplication of 5 and 3, (3 times 5 is 15) is a composite number; and these numbers, 5, and 3, are called *component parts.* Therefore,

1. *If the multiplier be a composite number;* multiply first by one of the component parts, and that product by the other; the last product will be the answer sought.

EXAMPLES.

1. Multiply 6 7 by 1 5

OPERATION.

```
  6 7
    5 one of the component parts.
 ----
 3 3 5
    3 the other component part.
 ----
1 0 0 5 Product of 67 mult. by 15.
```

2. Multiply 367 by 48, Product, 17616.

OPERATION.

Consider first what two numbers mutiplied together will produce 48; that is, what are the component parts of 48?—Answer, 6 and 8 (6 times 8 is 48) therefore multiply 367 first by one of the component parts, and the product thence arising by the other; the last product will be the answer sought.

3. Mult. 583 by 56. *Prod.* 32648.

OPERATION.

4. Mult. 1086 by 72. *Prod.* 78192.

OPERATION.

2. "*When there are cyphers on the right hand of either the multiplicand or*
"*multiplier, or both,* neglect those cyphers; then place the significant fig-
"ures under one another, and multiply by them only; add them together
"as before directed, and place to the right hand as many cyphers as there
"are in both the factors."

EXAMPLES.

1. Multiply 65430 by 5200.

OPERATION.

```
      6 5 4 3 0
          5 2 0 0
      ----------
3 4 0 2 3 6 0 0 0
```

Here in the multiplication of 65430 by 5200, the cyphers are seen neglected, and regard paid only to the significant figures. To the product are annexed 3 cyphers; equal to the number of cyphers neglected in the factors.

2. Mult. 36500
By 730

Prod. 26645000

3. Mult. 78000 by 600.
Product, 46800000.

3. *When there are cyphers between the significant figures of the multiplier*, omit the cyphers and multiply by the significant figures only, placing the first figure of each product directly under the figure by which you multiply, and adding the products together, the sum of them will be the product of the given numbers.

EXAMPLES.

1. Mult. 154326 by 3007.

OPERATION.

```
     1 5 4 3 2 6
         3 0 0 7
     -----------
   1 0 8 0 2 8 2
 4 6 2 9 7 8
 ---------------
 4 6 4 0 5 8 2 8 2
```

In this example, the cyphers in the multiplier are neglected, and 154326 multiplied only by 7 and by 3, taking care to place the figure in each product directly under the figure from which it was obtained.

2.

```
 3 4 5 7
   3 0 2
-------------
1 0 4 4 0 1 4
```

3.

```
4 8 9 7 6 8 5 0
    4 0 0 0 3 0
---------------






---------------------------
1 9 5 9 2 2 0 9 3 0 5 5 0 0
```

4. *When the Multiplier is* 9, 99, *or any number of* 9's, annex as many cy pher's to the Multiplicand, and from the number thus produced, subtract the multiplicand, the remainder will be the product.

EXAMPLES.

1. Mult. 6547 by 999

OPERATION.

```
6 5 4 7 0 0 0
      6 5 4 7
-------------
6 5 4 0 4 5 3
```

Write down the Multiplicand, place as many cyphers at the right hand as there are 9's in the Multiplier for a *minuend;* underneath write again the multiplicand for a *Subtrahend*, subtract, and the remainder is the product of 6547 multiplied by 999.

2.

6473 } *Product*, 640827
99 }

3.

7021 } *Product*, 695079
99 }

4.

5 3 8 4 9 7 6 } *Product*, 53844375024
9 9 9 9 }

SUPPLEMENT TO MULTIPLICATION.

QUESTIONS.

1. What is Simple Multiplication?
2. How many numbers are required to perform that operation?
3. Collectively or together, what are the given numbers called?
4. Separately, what are they called?
5. What is the result, or number sought, called?
6. In what order must the given numbers be placed for multiplication?
7. How do you proceed when the multiplier is less than 12?
8. When the multiplier exceeds 12 what is the method of procedure?
9. What is a composite number?
10. What is to be understood by the component parts of any number?
11. How do you proceed when the multiplier is a composite number?
12. When there are cyphers on the right hand of the multiplier, multiplicand, either or both, what is to be done?
13. When there are cyphers between the significant figures of the multiplier, how are they to be treated?
14. When the multiplier consists of 9's how may the operation be contracted?
15. How is Multiplication proved?
16. By what method do you proceed in casting out the 9's from any number?
17. How is Multiplication proved by casting out the 9's?

EXERCISES.

1. What sum of money must be divided between 27 men, so that each may receive 115 dollars. *Ans.* 3105.

NOTE. The scholar's business in all questions for Arithmetical operations, is wholly with the numbers given; these are never less than *two*; they may be more, and these numbers *in one way or another*, are always to be made use of to find the answer. To these, therefore, he must direct his attention, and carefully consider what is proposed by the question to be known.

2. An army of 10700 men having plundered a city, took so much money, that when it was shared among them, each man received 46 dollars; what was the sum of money taken? *Ans.* 492200.

3. There were 175 men employed to finish a piece of work, for which each man was to receive 13 dollars; what did they all receive? *Ans.* 2275.

4. Suppose a certain town contains 145 houses, each house two families, and each family 6 inhabitants; how many would be the inhabitants of that town? *Ans.* 1740.

5. If a man earn 2 dollars per week, how much will he earn in 5 years, there being 52 weeks in a year? *Ans.* 520 dolls.

6. How much wheat will 36 men thrash in 37 days, at 5 bushels per day each man? *Ans.* 6660 bushels.

7. If the price of wheat be 1 dollar per bushel, and 4 bushels of wheat make 1 barrel of flour, what will be the price of 175 barrels of flour? *Ans.* 700 dolls.

§ 4. SIMPLE DIVISION.

SIMPLE DIVISION teaches, having two numbers given of the same denomination, to find how many times one of the given numbers contains the other. Thus, it may be required to know how many times 21 contains 7; the answer is 3 times. The larger number (21) or number to be divided, is called the *Dividend;* the lesser number (7) or number to divide by, is called the *Divisor;* and the answer obtained, (3) the *Quotient.*

After the operation, should there be any thing left of the Dividend, it is called the *Remainder.* This part, however, is uncertain; sometimes there is no remainder. When it does happen it will always be less than the divisor, if the work be right, and of the same name with the dividend.

RULE.

1. "Assume as many figures on the left hand of the dividend as contain "the divisor once or oftener; find how many times they contain it, and "place the answer as the highest figure of the quotient.

2. "Multiply the divisor by the figure you have found, and place the "product under that part of the dividend from which it was obtained.

3. "Subtract the product from the figures above it.

4. "Bring down the next figure of the dividend to the remainder and "divide the number it makes up as before."

When you have brought down a figure to the remainder, if the number it makes up be still less than the divisor, a cypher must be placed in the quotient, and another figure brought down.

EXAMPLES.

1. Divide 127 by 5.

```
Divisor.  Dividend.  Quotient.
   5)     1 2 7     (2 5
          1 0
          -----
            2 7
            2 5
            ---
              2  Remainder.
```

The parts in Division are to stand thus, the dividend in the middle, the divisor on the left hand, the quotient on the right, with a half parenthesis separating them from the dividend.

Proceed in this operation thus—It being evident that the divisor (5) cannot be contained in the first figure (1) of the dividend, therefore assume the two first figures (12) and inquire how often 5 is contained in 12; finding it to be 2 times, place 2 in the quotient, and multiply the divisor by it, saying 2 times 5 is 10, and place the sum (10) directly under 12 in the dividend. Subtract 10 from 12 and to the remainder (2) bring down the next figure (7) at the right hand, making with the remainder 27. Again inquire how many times 5 in 27; 5 times; place 5 in the quotient, multiply the divisor (5) by the last quotient figure (5) saying 5 times 5 is 25, place the sum (25) under 27, subtract and the work is done. Hence it appears that 127 contains 5, 25 times, with a remainder of 2, which was left after the last subtraction.

This Rule, perhaps at first will appear intricate to the young student, although it is attended with no difficulty. His liability to errors will chiefly arise from the diversity of proceedings. To assist his recollection, let him notice that

The steps of Division are four:
1. Find how many times, &c.
2. Multiply.
3. Subtract.
4. Bring down.

It is sometimes practised to make a point (.) under the figures in the dividend, as they are brought down, in order to prevent mistakes.

When the divisor is a large number, it cannot always certainly be known how many times it may be taken in the figures which are assumed on the left hand of the dividend till after the first steps in division are gone over, but the learner must try so many times as his judgment may best dictate, and after he has multiplied, if the product be greater than the number assumed, or that number in which the divisor is taken, then it may always be known that the quotient figure is too large; if after he has multiplied and subtracted, the remainder be greater than the divisor, then the quotient figure is not large enough, he must then suppose a greater number of times, and proceed again. This at first may occasion some perplexity, but the attentive learner after some practice, will generally hit on the right number.

2. Let it be required to divide 7012 by 52.

OPERATION.

```
Divisor. Dividend. Quotient.
 5 2) 7 0 1 2 (1 3 4
      5 2
      ---
      1 8 1
      1 5 6
      -----
        2 5 2
        2 0 8
        -----
          4 4 Remainder.
```

In this operation it is left for the scholar to trace the steps of procedure without having them particularly pointed out to him by words.

PROOF.

Division may be proved by multiplication.

RULE.

"Multiply the Divisor and Quotient together, and add the remainder, if "there be any to the product; if the work be right, the sum will be equal "to the dividend."

Take the last Example.

```
The Quotient was  1 3 4 } Multiply them together.
Divisor             5 2 }
                -------
                  2 6 8
                6 7 0
                    4 4 Remainder added.
                -------
                7 0 1 2 Equal to the dividend.
```

Another and more expeditious way of proving Division is

By casting out the 9's.

Cast out the 9's from the Divisor and the Quotient, multiply the results and to the product, add the remainder if any after division; from the sum of these cast out the 9's, also cast out the 9's from the Dividend, and if the two last results agree, the work is supposed to be right.

3. Divide 17354 by 86.

OPERATION. PROOF.

Divis.	*Divid.*	*Quot.*		
86)1 7 3 5 4(2 0 1	———	9's out of (*Divis.*) 86 *Rem.* 5	Multiplied	
		(*Quot.*) 201 *Rem.* 3	together.	
1 7 2 . .		15		
1 5 4		Remainder 68 added.		
8 6				
6 8 *Rem.*		9's out of 83 *Rem.* 2	agreeing	
		9's out of (*Divid.*) 17354 *Rem.* 2	together	

4. Divide 153598 by 29. *Quotient* 5296. Rem. 14.

5 Divide 8893810 by 235. *Quot.* 37846.

6. Divide 30114 by 63. *Quotient* 478

7. Divide 9302688 by 648 *Quot.* 14356.

8 Divide 974932 by 365. *Quotient* 2671. *Rem.* 17

9. Divide 5221580 by 68705. *Quot.* 76.

10. Divide 3228242 dollars equally among 563 men; how many dollars must each man receive? *Ans.* 5734.

From a view of the question, it is evident, that the dollars must be divided into as many parts as there are men to receive them; consequently, the number of dollars must be made the *dividend*, and the number of men the *divisor;* the quotient will then shew how many dollars each man must receive.

11. How many times does 1030603615 contain 3215? *Ans.* 320561 *times.*

Contractions and varieties in Division.

I. *When the divisor does not exceed* 12, the operation may be performed without setting down any figures excepting the quotient, by carrying the computation in the mind. The units which would remain after subtracting the product of the quotient figure and the divisor from the figures assumed of the dividend, must be accounted so many *tens*, and be supposed to stand at the left hand of the next figure in the dividend, then consider again how often the divisor may be had in the sum of them. Proceed in this way till all the figures in the dividend have been divided. This is called *short division.*

EXAMPLES.

1. Divide 732 by 3.

OPERATION.

3) 7 3 2
―――――
2 4 4 *Quotient.*

Here I say, how often 3 in 7, knowing it to be 2 times, I place 2 in the quotient, then considering that the quotient figure (2) and the divisor (3) multiplied together would be 6, and that this product (6) subtracted from 7 in the dividend, would leave 1, I then consider this remainder (1) as standing at the left hand of the next figure (3) of the dividend, which together make 13. I now say how many times 3 in 13—4 times, therefore I place 4 in the quotient, which multiplied into the divisor (3) would be 12, and 12 subtracted from 13 would leave 1, which considered as standing at the left hand of the next or last figure (2) of the dividend would make 12; again, how many times 3 in 12—4 times—I then place 4 in the quotient, which multiplied into the divisor (3) is 12, this product (12) I consider as subtracted from 12, I find there will be no remainder, and the work is done.

2. Divide 37426 by 7.

OPERATION.

7) 3 7 4 2 6
―――――
Quot. 5 3 4 6 *Rem.* 4

Here I say how often 7 in 37? 5 times and two remain; then how often 7 in 24? 3 times and 3 remain; how often 7 in 32? 4 times and 4 remain; lastly, how often 7 in 46? 6 times, 4 remain.

3. Divide 310658023162	by 4	*Quot.*	77664505790	*Rem.* 2
4. - - - 621075214621	- 5	-	124215042924	- - 1
5. - - - 213524031628	- 6	-	35587338604	- - 4
6. - - - 306452706527	- 8	-	38306588315	- - 7
7. - - - 546321406968	- 9	-	60702378552	

II. *When there are cyphers at the right hand of the divisor*, cut them off, also cut off an equal number of figures from the right hand of the dividend and place these figures at the right hand of the remainder.

EXAMPLES.

1. Divide 6203946 by 5700.

OPERATION.

```
57 | 00)62039 | 46(1088
       57...
       -----
        503
        456
        ---
        479
        456
        ----
        2346
```

Here are two cyphers on the right hand of the divisor which I cut off, also I cut off two figures (46) from the dividend and to the right hand of the remainder after the last division (23) I place the figures cut off from the dividend (46) which make the whole remainder 2346.

2. Divide 379432 by 6500 *Quot.* 58. *Rem.* 2432

3. Divide 2764503721 by 83000. *Quot.* 33307. *Rem.* 22721.

4. *When the divisor is* 10, 100, 1000, *or* 1, *with any number of cyphers annexed*, cut off as many figures on the right hand of the dividend as there are cyphers in the divisor; the figures which remain of the dividend compose the *quotient*; those cut off, the *remainder*.

EXAMPLES.

1. Divide 1576 by 10.

OPERATION.

1 | 0) 1 5 7 | 6

Here we have one cypher in the divisor, therefore cut off one figure (6) from the dividend; what remains (157) is the *quotient*, and the figure cut off (6) the *remainder*.

2. Divide 3217 by 100. *Quot.* 32. *Rem.* 17.

3. Divide 76421795 by 1000. *Quot.* 76421. *Rem.* 795

SUPPLEMENT TO DIVISION.

QUESTIONS.

1. What is Simple Division ?
2. How many numbers must there be given to perform the operation ?
3. What are the given numbers called ?
4. How are they to stand for Division ?
5. How many steps are there in Division ?
6. What is the first ? the second ? the third ? the fourth ?
7. What is the result or answer called ?
8. Is there any other or uncertain part pertaining to Division ? What is it called ?
9. Of what name or kind is the remainder ?
10. What is short Division ?
11. When there are cyphers at the right hand of the divisor, what is to be done ?
12. What do you do with figures cut off from the dividend when there are cyphers cut off from the divisor ?
13. When the divisor is 10, 100, or 1 with any number of cyphers annexed, how may the operation be contracted ?
14. How many ways may Division be proved ?
15. How is Division proved by Multiplication ?
16. How may Division be proved by casting out the 9's ?

EXERCISES.

1. Suppose an estate of 36582 dollars to be divided among 13 sons, how much would each one receive ? *Ans.* 2814 *dolls.*

2. An army of 15000 men having plundered a city, and took 2625000 dollars, what was each man's share ? *Ans.* 175 *dolls.*

3. A certain number of men were concerned in the payment of $18950 and each man paid 25 dollars, what was the number of men? *Ans.* 758.

4. If 7412 eggs be packed in 34 casks, how many in a cask? *Ans.* 218.

5. A farm of 375 acres is let for 1125 dollars; how much does it pay per acre? *Ans.* 3 *dolls.*

6. A field of 27 acres produces 675 bushels of wheat; how much is that per acre? *Ans.* 25 *bushels.*

7. Supposing a man's income to be 2555 dollars a year: how much is that per day, there being 365 days in a year *Ans.* 7 *dolls.*

8. What number must I multiply by 13, that the product may be 871? *Ans.* 67.

§ 5. COMPOUND ADDITION.

COMPOUND ADDITION is the adding of numbers which consist of articles of different value, as pounds, shillings, pence, and farthings, called *different denominations;* the operations are to be regulated by the value of the articles, which must be learned from the Tables.

RULE FOR COMPOUND ADDITION.

1. Place the numbers so that those of the same denomination may stand directly under each other.

2. Add the first column or denomination together, and carry for that number which it takes of the same denomination to make 1 of the next higher. Proceed in this manner with all the columns, till you come to the last, which must be added, as in Simple Addition.

1. OF MONEY.

TABLE.

4 Farthings *qr.*	make one	Penny, *marked d.*
12 Pence		Shilling, *s.*
20 Shillings		Pound, £.

EXAMPLES.

1. What is the sum of £61. 17*s.* 5*d.*——£13. 3*s.* 8*d.*——and of £5. 16*s.* 11*d.* when added together?

OPERATION.

£.	s.	d.	
61	17	5	Those numbers of the same denomination placed under each other, as the rule directs.
13	3	8	
5	16	11	
80	18	0	

I begin with the right hand column or that of pence, and having added it, find the sum of the numbers therein contained to be 24; now as 12 of this denomination make one of the next higher, or in other words 12 pence make one shilling, therefore in this or in the column of pence I must carry for 12; I now inquire how often 12 is contained in 24, the sum of the first column or that of pence; knowing it to be 2 times and nothing over, I set down 0 under the column of pence, and carry 2 to that of shillings, to be added into the second column, saying, 2 I carry to 6 are 8, and 3 are 11, and 7 are 18, and 10 to 18 are 28, and ten again are 38 (for so each figure in ten's place must be reckoned, 1 in that place, being equal in value to 10 units.) Now as 20 shillings make one pound, therefore in the column of shillings, I carry for 20; I then inquire how often 20 in 38? once, and 18 remains; therefore, I set down directly under the column of shillings 18, what 38 contains more than 20, and for the even 20 carry 1 to pounds or the last column, which is to be added after the manner of Simple Addition.

Note.—The method of proof for Compound Addition is the same as that of Simple Addition.

2.

£.	s.	d.	qr
18	4	11	1
26	15	3	0
8	1	7	3

3.

£.	s.	d.	qr.
371	15	6	2
5	7	4	0
68	3	2	1
7	0	5	3

4. Supposing a man goes a journey, and on the 1st day,

1802. May 14,	Pays for a dinner - - - - - - - - -	£0	1	6
	- for oats for his horse - - - -	0	0	6
	- for shoeing - - - - - - - - -	0	1	2
15,	- for supper and lodging - - - -	0	2	0
	- for horse keeping - - - - - -	0	1	10
	- for toll - - - - - - - - - -	0	1	6
	- for breakfast - - - - - - - -	0	2	0
	- to the barber for dressing - - -	0	1	6
	- for dinner again and refreshment -	0	3	5

What were the gentleman's expenses?

5. Suppose I am indebted £. s. d.

To A. Thirty-two pounds fourteen shillings and ten pence.

— B. Forty-one pounds six shillings and eight pence.

— C. Seventy-five pounds eight shillings.

— D. Three pounds and nine pence.

What is the sum I owe? *Ans.*

6. A man purchases cattle; one yoke of oxen for £14 11 6; four cows, for £18 19 7; and other stock to the amount of £21 5; what was the amount of the cattle purchased? *Ans.* £54 16*s.* 1*d.*

2. OF TROY WEIGHT.

By Troy Weight are weighed gold, silver, jewels, electuaries and liquors.

TABLE.

24 grains *grs.*	make one	Pennyweight, *marked pwt.*
20 Pennyweights	make one	Ounce, *oz.*
12 Ounces	make one	Pound, *lb.*

EXAMPLES.

1

lb.	*oz.*	*pwt.*	*grs.*
70	10	13	4
3	9	7	16
28	0	0	5
7	3	6	2

Because 24 grains make a pennyweight, you carry one to the pennyweight column for every 24 in the sum of the column of grains; because 20 pennyweights make one ounce, you carry for 20 in pennyweights, and because 12 ounces make one pound, you carry for 12 in the ounces. This is called carrying according to the value of the higher place.

lb.	*oz.*	*pwt.*
151	7	19
6	5	6
28	0	14
	3	7

2.

lb.	*oz.*	*pwt.*	*grs.*
	7	14	23
	2	0	6
	11	13	5
	10	12	7

Note.—The fineness of gold is tried by fire, and is reckoned in *carats*, by which is understood the 24th part of any quantity; if it lose nothing in the trial, it is said to be 24 carats fine; if it lose 2 carats, it is then 22 carats fine, which is the standard for gold.

Silver which abides the fire without loss is said to be 12 ounces fine.—The standard for silver coin is 11 oz. 2 pwts. of fine silver, and 18 pwts. of copper melted together.

3. OF AVOIRDUPOIS WEIGHT.

By Avoirdupois weight are weighed all things of a coarse and drossy nature, as tea, sugar, bread, flour, tallow, hay, leather, and all kinds of metals, except gold and silver.

TABLE.

16 Drams *dr.*	make one	Ounce,	*marked* *oz.*
16 Ounces		Pound,	*lb.*
28 Pounds		Quarter of a hund. weight,	*qr*
4 Quarters		100 weight, or 112 pounds,	*cwt.*
20 Hundred weight		Ton,	*T.*

EXAMPLES.

1.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
186	3	2	25	11	8
4	17	0	23	7	6
9	8	3	7	2	5
2	3	1	16	5	11

2.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
801	3	2	25	11	8
7	19	3	14	5	6
86	2	0	6	0	15
3	7	1	0	6	4

Note.—" 175 Troy ounces are precisely equal to 192 Avoirdupois Ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1 lb. Troy=5760 grains, and 1 lb. Avoirdupois=7000 grains."

4. OF TIME.

TABLE.

60 Seconds *s.*	make one	Minute, *marked m.*
60 Minutes		Hour *h.*
24 Hours		Day, *d.*
7 Days		Week, *w.*
4 Weeks		Month, *mo.*
13 Months, 1d. & 1h.		* Julian Year, *Y.*

EXAMPLES.

1.

Y.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
16	10	3	6	23	57	43
28	7	2	5	16	28	32
39	6	1	3	17	38	11
87	4	0	1	14	15	17

2.

Y.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
89	11	3	6	22	45	36
36	10	2	5	6	55	44
87	2	1	0	11	22	33
36	4	3	3	5	8	7

The number of days in each Calendar month may be remembered by the following verse :

Thirty days hath September, April, June and November;
February twenty-eight alone ; all the rest have thirty-one.

* " The civil Solar year of 365 days being short of the true by 5h. 48m. 57s. occasioned the beginning of the year to run forward through the season nearly one day in four years · on this account Julius Cæsar ordained that one day should be added to February every fourth year, by causing the 24th day to be reckoned twice : and because this 24th day was the sixth (sextilis) before the kalends of March, there were in this year two of these sextiles, which gave the name of Bissextile to this year, which being thus corrected, was, from thence called the Julian year."

5. OF MOTION.

TABLE.

60 Seconds	make one	Prime Minute, *marked* ″ ′
60 Minutes		Degree, °
30 Degrees		Sign, s.
12 Signs, or 360 Degrees		The whole great circle of the Zodiac.

EXAMPLES.

1.

°	′	″
25	17	18
17	49	56
6	35	24
10	17	16

2.

s.	°	′	″
9	8	55	44
1	26	44	55
8	18	36	12
1	9	33	22

6. OF CLOTH MEASURE.

TABLE.

2 Inches, one fifth *in.*	make one	Nail, *marked na.*
4 Nails, or 9 inches		Quarter of a yard, *qr.*
4 Quarters of a yard or 36 inches		Yard, *yd.*
3 Quarters of a yard, or 27 inches		Ell Flemish *E. Fl.*
5 Quarters of a yard or 45 inches		Ell English *E. E.*
6 Quarters of a yard, or 54 inches		Ell French *E. Fr.*
4 Quarters 1 inch and 1 fifth, or 37 inches and one fifth		Ell Scotch *E. Sc.*
3 Quarters and two thirds		Spanish Var.

EXAMPLES.

1.

Yds.	*qrs.*	*n.*
614	3	3
36	1	2
7	0	1
15	3	2

2.

E. E.	*qr.*	*n.*
19	3	2
56	1	3
7	2	2
18	2	0

7. OF LONG MEASURE.

By Long Measure are measured distances, or any thing where length is considered without regard to breadth.

TABLE.

3 Barley corns, *bar.*	make one	Inch, *marked*	*in.*
12 Inches		Foot,	*ft.*
3 Feet		Yard,	*yd.*
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet		Rod, Perch or Pole,	*pol.*
40 Poles		Furlong,	*fur.*
8 Furlongs		Mile,	*mile.*
$69\frac{1}{2}$ Statute miles, *nearly,*		Degree of a great Circle.	
360 Degrees		A great Circle of the Earth.	

EXAMPLES.

1.

Deg.	*mi.*	*fur.*	*pol.*	*ft.*	*in.*	*bar.*
168	57	7	26	15	11	2
124	53	6	18	7	6	1
79	36	1	7	9	10	0
4	7	3	0	3	2	1

2.

Deg.	*mi.*	*fur.*	*pol.*	*ft.*	*in.*
13	56	5	13	8	1
49	18	1	27	16	2
267	12	3	16	9	0
29	8	0	5	3	1

8. OF LAND OR SQUARE MEASURE.

By Square Measure are measured all things that have length and breadth.

TABLE.

144 Inches	make one	Square Foot.
9 Feet	make one	—— Yard.
$30\frac{1}{4}$ Yards, or $272\frac{1}{4}$ Feet	make one	—— Pole.
40 Poles	make one	—— Rood.
4 Roods 160 rods or 4840 yards	make one	—— Acre.
640 Acres	make one	—— Mile.

EXAMPLES.

Acres.	*rood.*	*pol.*	*ft.*	*in*
376	3	36	93	121
568	1	27	58	76
247	2	35	61	24

9. OF SOLID MEASURE.

By Solid Measure are measured all things that have length, breadth and thickness.

TABLE.

1728 Inches	make one	Foot.
27 Feet	make one	Yard.
40 Feet of round timber or 50 feet of hewn timber	make one	Ton or load.
128 Solid feet, i. e. 8 in length, 4 in breadth and 4 in height	make one	Cord of wood.

EXAMPLES.

1.			2.		
Ton.	*ft.*	*in.*	*Cord.*	*ft.*	*in*
65	37	229	39	118	1021
19	26	1207	4	56	137
36	17	54	18	72	659
57	33	6	29	86	124

10. OF WINE MEASURE.

By Wine Measure are measured Rum, Brandy, Perry, Cider, Mead, Vinegar and Oil.

TABLE.

2 Pints *pts.*	make one	Quart,	*marked*	*qt.*
4 Quarts	make one	Gallon,		*gal.*
10 Gallons	make one	Anchor of Brandy,		*anc.*
18 Gallons	make one	Runlet,		*run.*
$31\frac{1}{2}$ Gallons	make one	Half Hogshead.		$\frac{1}{2}$*hhd.*
42 Gallons	make one	Tierce,		*tier.*
63 Gallons	make one	Hogshead,		*hhd.*
2 Hogsheads	make one	Pipe or Butt,		*P. or B.*
2 Pipes	make one	Tun.		*T*

EXAMPLES.

1.

Hhd.	*gal.*	*qts.*	*pts.*
39	52	3	1
16	27	1	0
35	12	0	1
29	38	2	0

2.

T.	*hhd.*	*gal.*	*qts.*	*pts*
86	2	53	3	1
35	1	36	1	0
17	0	29	2	1

N. B. A *Pint* Wine Measure is $28\frac{7}{8}$ cubic inches.

11. OF ALE OR BEER MEASURE.

TABLE.

2 Pints	make one	Quart, *marked* *qts*
4 Quarts		Gallon, *gal.*
8 Gallons		Firkin of Ale in London, A. *fir.*
8½ Gallons		Firkin of Ale or Beer.
9 Gallons		Firkin of Beer in London, B. *fir.*
2 Firkins		Kilderkin, *kill.*
2 Kilderkins		Barrel, *bar.*
1½ Barrels, or 54 gallons		Hogshead of Beer, *hhd.*
2 Barrels		Puncheon, *pun.*
3 Barrels, or 2 hogsheads		Butt, *butt.*

EXAMPLES.

1.

hhd.	*gall.*	*qts.*
327	48	2
28	51	3
173	24	1
27	16	0

2.

B. fir.	*gal.*	*qts.*
23	6	2
45	2	3
98	7	1
36	8	0

N. B. A *Pint* Beer Measure, is 35¼ cubic inches.

12. OF DRY MEASURE.

By Dry Measure are measured all Dry Goods, such as Corn, Wheat, Seed, Fruit, Roots, Salt, Coal, &c.

TABLE.

2 Pints	make one	Quart, *marked* *qts.*
2 Quarts		Pottle, *pot.*
2 Pottles		Gallon, *gal.*
2 Gallons		Peck, *pk.*
4 Pecks		Bushel, *bush.*
2 Bushels		Strike, *str.*
2 Strikes		Coom, *co.*
2 Cooms		Quarter, *qr.*
4 Quarters		Chaldron, *ch.*
4½ Quarters		Chaldron in London.
5 Quarters		Wey, *wey*
2 Weys		Last, *last*

EXAMPLES.

1.

bush.	*pk.*	*qt.*	*pt.*
27	2	6	1
18	8	7	0
20	0	1	1
10	1	3	0

2.

Ch.	*bush.*	*pk.*	*qts.*
37	16	2	5
26	28	3	7
18	12	1	0
17	25	3	6

N. B. A *gallon*, Dry Measure, contains 268¼ cubic inches

The following are denominations of things counted by the "Tale."

12 Particular things make 1 Dozen,
12 Dozen . . . . . . 1 Gross,
12 Gross or 144 doz. . 1 great Gross.

ALSO,

20 Particular things make 1 Score.

Denominations of Measures not included in the Tables.

6 Points make 1 Line,
12 Lines . . Inch,
4 Inches . . Hand,
3 Hands . . Foot,
66 Feet, or 4 Poles, a Gunter's Chain,
3 Miles . . League.

A Hand is used to measure Horses. A Fathom to measure depths.
A League in reckoning distances at Sea.
N. B A Quintal of Fish weighs 1 cwt. Avoirdupois.

§ 6. COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches to find the difference between any two sums of diverse denominations.

RULE FOR COMPOUND SUBTRACTION.

" Place those numbers under each other, which are of the same denom-
" ination, the less being below the greater; begin with the least denomina-
" tion, and if it exceed the figure over it, borrow as many units as make
" one of the next greater; subtract it therefrom; and to the difference
" add the upper figure, remembering always to add *one* to the next supe-
" rior denomination for that which you borrowed.

Proof.—In the same manner as Simple Subtraction.

1. OF MONEY.

1. Supposing a man to have lent £185 10*s*. 7*d*. and to have received again of his money, £93 15*s*. how much remains due?

OPERATION.

1.

	£.	*s*.	*d*.
Lent	1 8 5	1 0	7
Received	9 3	1 5	0
Due	9 1	1 5	7
Proof	1 8 5	1 0	7

2.

	£.	*s*.
From	3 1 0	
Take	8 5	1 5

3.

	£.	*s*.	*d*.
Lent	6 3 7 1	7	8
Received at sundry times.	1 6 3	2	5
	7 8	4	
	1 9	15	11
	1 3 9	6	8
	3 2 6 1	1	4
Received in all			
Yet due			

The sum of the several payments must first be added together, and the amount subtracted from the sum lent.

4. From £39 7*s*. 6*d*. 1 *qr*. take £7 13*s*. 11½*d*. and what will remain?

Ans. £31 13*s*. 6¾*d*.

5. A certain man sold a lot of land for £735 11*s.* 6*d*: he received at one time £61 5*s*: at another time, £195 13*s.* 11*d.* how much is there yet due? *Ans.* £478 12*s.* 7*d.*

2. OF TROY WEIGHT.

1.

	lb.	oz.	pwt.	grs.
From	76	8	16	13
Take	3	9	17	6
Remains				
Proof.				

2.

lb.	oz.	pwt.
7	3	5
2	8	9

3. OF AVOIRDUPOIS WEIGHT.

1.

lb.	oz.	dr.
9	15	5
5	6	7

2.

T.	cwt.	qr.	lb.	oz.	dr
6	11	1	14	7	3
1	5	1	16	9	8

4. OF TIME.

1.

Y.	mo.	w.	d.	h.	m.	s.
39	6	3	6	20	44	55
16	9	1	2	18	59	57

5. OF MOTION.

1.

°	′	″
16	27	33
8	34	23

2.

s.	°	′
6	8	51
3	9	57

6. OF CLOTH MEASURE.

1.

Yds.	*qr.*	*n.*
27	1	2
16	1	3

2.

E. E.	*qr.*	*n.*
26	2	1
17	3	2

7. OF LONG MEASURE.

1.

Deg.	*mi.*	*fur.*	*p.*	*yds*	*ft.*	*in*	*bar.*
56	13	5	26	2	1	8	1
17	15	2	27	1	2	9	1

8. OF LAND OR SQUARE MEASURE.

1.

A.	*R.*	*pol.*
17	1	17
16	1	16

2.

pol.	*ft.*	*in.*
18	16	11
10	201	130

9. OF SOLID MEASURE.

1.

Tons.	ft.	in.
45	29	186
19	34	1237

2.

Cords.	ft.	in.
68	23	810
6	127	1529

10. OF WINE MEASURE.

1.

Hhd.	gal.	qts.
66	31	2
17	33	3

2.

Tun.	Hhd.	gal.
75	1	16
24	1	43

11. OF ALE AND BEER MEASURE.

1.

Hhd.	gal.	qts.
89	19	2
37	25	3

2.

Butt.	hhd.	gal.
63	1	16
29	1	10

12. OF DRY MEASURE.

1.

Bu.	pk.	qts.
61	1	2
5	1	4

2.

Chal.	bu.	pk.
171	18	1
76	22	2

THE

SCHOLAR'S ARITHMETIC.

OBSERVATIONS.

THE Scholar has now surveyed the ***ground work*** of Arithmetic. It has before been intimated that the only way in which numbers can be affected, is by the operations of *Addition, Subtraction, Multiplication* and *Division.* These rules have now been taught him, and the exercises in a supplement to each, suggest their use and application to the purposes and concerns of life. Further, the ***thing needful***, and that which distinguishes the Arithmetician, is to know how to proceed by application of ***these four rules*** to the solution of any arithmetical question. To afford the scholar this knowledge is the object of all succeeding rules.

SECTION II.

RULES ESSENTIALLY NECESSARY FOR EVERY PERSON TO FIT AND QUALIFY THEM FOR THE TRANSACTION OF BUSINESS.

These are ten: Reduction, Fractions, Federal Money, Exchange, Interest, Compound Multiplication, Compound Division, Single Rule of Three, Double Rule of Three, and Practice.*

A thorough knowledge of these rules is sufficient for every ordinary occurrence in life. Short of this a person in any kind of business, will be liable to repeated embarrassments. It is the extreme usefulness of these rules which commends them to the attention of every Scholar.

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* FRACTIONS are taken up here no further than is necessary to shew their signification, and to illustrate the principles of FEDERAL MONEY.

§ 1. REDUCTION.

"REDUCTION teaches to bring or exchange numbers of one denomination to others of different denominations, retaining the same value."

IT IS OF TWO KINDS.

1. *When high denominations are to be brought into lower,* as pounds into shillings, pence and farthings; it is then called REDUCTION DESCENDING, and is performed by *Multiplication.*

2. *When lower denominations are to be brought into higher,* as farthings into pence, or into pence, shillings and pounds; it is then called REDUCTION ASCENDING, and is performed by *Division.*

REDUCTION DESCENDING.

RULE.

MULTIPLY the highest denomination by that number which it takes of the next less to make one of that greater; so continue to do till you have brought it as low as your question requires.

PROOF—"Change the order of the question, and divide your last product by the last multiplier, and so on."

EXAMPLES.

1. In £17 13*s.* 6*d.* 3*qrs.* how many farthings?

OPERATION.

£.	s.	d.	qrs.	
17	13	6	3	
20				*Shillings in a pound.*
353				*Shillings in* £17 13*s.*
12				*Pence in a Shilling.*
4242				*Pence in* £17 13*s.* 6*d.*
4				*Farthings in a penny.*
A. 16971				*Farthings.*

In this example, the highest denomination is pounds, the next less, is shillings, and because 20 shillings make *one pound,* therefore, I multiply £17 by 20, increasing the product by the addition of the given shillings (13) which it must be remembered, must always be done in like cases; then because 12 pence make *one shilling,* I multiply the shillings (353) by 12, adding in the given pence (6*d.*) lastly, because 4 farthings make one penny, I multiply the pence (4242) by 4, and add in the given farthings (3*qrs.*) I then find that in £17 13*s.* 6*d.* 3*qrs.* there are 16971 farthings.

PROOF.

4) 16971 3*qrs.*

12) 4242 6*d.*

2|0) 35|3 13*s.*

£17

To prove the above question, change the order of it, and it will stand thus: in 16971 farthings, how many pounds?

Divide the last product by the last multiplier, the remainder will be farthings. Proceed in this way till all the steps of the operation have been retraced back; the last quotient with the remainders will be proof of the accuracy of the operation if they agree with the sum given in the question.

2. In £7 14s. 6d. 1qr. how many farthings? *Ans.* 7417*qrs.*

3. In £7 6s. 4d. how many pence? *Ans.* 1756*d.*

4. In 29 guineas, at 28s. each, how many farthings? *Ans.* 38976 *qrs.*

5. In £173 15s. how many six-pences? *Ans.* 6950.

6 In 12 crowns at 6s. 7d. how many pence and farthings? *Ans.* 948*d.* 3792*qrs.*

7. In 671 eagles, at 10 dolls. each, how many shillings, three-pences, pence, and farthings? *Ans.* 40260*s.* 161040 *three-pences*, 483120 *pence*, and 1932480*qrs.*

REDUCTION ASCENDING.

RULE.

Divide the lowest denomination given by that number which it takes of the same to *make one* of the next higher, and so continue to do till you have brought it into the denomination which your question requires.

EXAMPLES.

1. In 16971 farthings how many pounds?

OPERATION.

Farthings in a penny	4)16971 3*qrs.*
Pence in a shilling	12)4242 6*d.*
Shillings in a pound	2|0)35|3 13*s.*
	£17

Ans. £17 13*s.* 6*d.* 3*qrs.*

Reduction descending and ascending reciprocally prove each other.

2. In 1765 pence, how many pounds? *Ans.* £7 7*s.* 1*d.*

3. In 38976 farthings how many guineas? *Ans.* 29

4. In 6950 sixpences, how many pounds? *Ans.* £173 15*s.*

5. In 3792 farthings, how many crowns? *Ans.* 12.

6. In 48960 farthings, how many pence, three-pences, six-pences and dollars?

Ans. 12240 *pence*, 4080 *three-pences*, 2040 *six-pences*, 170 *dollars.*

7. In 6952 three-pences, how many pistoles at 22s *Ans.* 79

REDUCTION ASCENDING & DESCENDING.

1. MONEY.

1. In 57 moidores, at 36s. each, how many dollars? *Ans.* $342.

In this question the first step will be to bring the moidores into shillings: lastly bring the shillings into dollars.

2. In 75 pistoles how many pounds? *Ans.* £82 10s.

3. In £73 how many guineas? *Ans.* 52 *guineas*, 4s.

4. In £63 and 5 guineas, how many dollars? *Ans.* $233 2s.

" *When it is required to know how many sorts of coin of different values and of equal number are contained in any number of another kind;* reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor; then reduce the money given into the same denomination for a dividend, and the quotient arising from the division will be the number required."

Note. Observe the same direction in weights and measures.

1. In 54 guineas, how many pounds, dollars and shillings of each an equal number?

OPERATION.

£1 is 20 shillings	54 guineas
1 dollar is 6 shillings	28 shillings is a guinea
1 shilling is 1 shilling	——
—	432
Divisor 27 shillings	108
	——
	Dividend 1512 shillings.

27)1512(56 of each; that is, 54 guineas include the value of one pound, one dollar, and one shilling 56 times.
135
——
162
162
——
000

2. In 172 moidores how many eagles, dollars and nine-pences of each the like number? *Ans.* 92 of each, and 68 nine-pences over.

3. In 237 guineas how many moidores, pistoles, pounds, and dollars each the like number? *Ans.* 79 of each

TROY WEIGHT.

1. In 4*lb.* 5*oz* and 16*pwts.* how many grains?

OPERATION.

lb.	*oz.*	*pwt.*
4	5	13

12 oz. in a pound.

53 ounces.
20 pwts. in an ounce.

1076 penny weights.
24 grains in 1 pwt.

4304
2152

Proof 24)25824 grains, the Ans.
20)1076 16 pwts.

12)53 5 oz.

4lb.

2. In 10*lb.* of silver, how many spoons, each weighing 5*oz.* 10 *pwts.*?
Ans. 21 *spoons, and* 90 *pwts. over.*

3 In 282240 grains of silver, how many pounds? *Ans.* 49.

PROOF

4. In 45681 grains of silver, how many pounds?

Answer 7lb 11oz. 3pwts. 9grs.

5. In 4560 grains of silver, now many tea spoons, each one ounce?

Ans. 9½ tea spoons.

PROOF.

3. *AVOIRDUPOIS WEIGHT.*

	Cwt.	*qr.*	*lb.*	*oz.*
1	In 67	1	13	11, how many drams?

4		PROOF.	
269		16)1931696	
28		16)120731	11*oz.*
2165		28)7545	13*lb*
538		4)269	1*qr.*
7545		67	*Cwt.*
16			
45281			
7545			
120731			
16			
724386			
120731			
1931696			

2. In 14048*oz.* how many hundred weight? *Ans.* 7*C.* 3*qrs.* 10*lb.*

3. In 470 boxes of Sugar, each 26*lb.* how many Cwt.?
Ans. 109*C.* 0*qrs.* 12*lb.*

4. In 17*Cwt.* 1*qr.* 6*lb* of Sugar, how many parcels, each 17*lb.*?
Ans. 114 *parcels*

4. TIME.

1. In 121812 seconds how many hours?

OPERATION.

$$\begin{array}{ll} 6|0)12181|2 & 12 \textit{ sec.} \\ \hline 6|0)203|0 & 50\textit{m.} \\ \hline \end{array}$$

Ans. 33*h.* 50*m.* 12*s.*

PROOF.

$$\begin{array}{rrr} H. & m. & s. \\ 33 & 50 & 12 \\ 60 & & \\ \hline 2030 & & \\ 60 & & \\ \hline 121812 & & \end{array}$$

2. Supposing a man to be 21 years old, how many seconds has he lived, allowing 365 days 6 hours to a year? *Ans.* 662709600 *seconds.*

3. How many minutes from the commencement of the war between America and England, April 19, 1775, to the settlement of a general peace, which took place, January 20, 1783? *Ans.* 4079160 *minutes.*

4. In 413280 minutes how many weeks? *Ans.* 41 weeks.

5. LONG MEASURE.

5. Reduce 16 miles to barley corns.

OPERATION.

```
   16 Miles.
    8
  ----
  128 Furlongs.
   40
  ----
 5120 Rods.
   5½*
 -----
25600
 2560
 -----
28160 Yards.
    3
 -----
84480 Feet.
   12
 -----
1013760 Inches.
      3
 -------
```

Answer, 3041280 *bar. corns.*

PROOF.

```
 3)3041280
12)1013760
  --------
   3)84480
  --------
 †11)28160
  --------
      2560
         2
  --------
 4|0)512|0
  --------
     8)128
  --------
  16 Miles.
```

† Divide by 11 for 5½ and multiply the quotient by 2. The reason is because 5½ reduced to *half yards* is 11.

* To multiply by *one half* (½) it is only to take half the multiplicand.

2. In 47520 feet how many leagues? *Ans.* 3 *leagues.*

3. How many times does the wheel which is 18 feet 6 inches in circumference, turn round in the distance of 150 miles?

Ans. 42810 times, and 180 inches over.

4. How many barley corns will reach round the Globe, it being 360 degrees? *Ans.* 4755801600.

6. *LAND OR SQUARE MEASURE.*

1. In 13 acres, 2 roods, how many poles?

OPERATION.

```
Ac.   R.
13    2
 4
--
54
40
--
```

Ans. 2160 *Poles.*

PROOF.

```
4|0)216|0
---------
  4)54
  ----
  13Ac. 2R.
```

2. In 2852 rods how many acres? *Ans.* 17*A.* 3*R.* 12*P*

7. *SOLID MEASURE.*

1. In 1296000 solid inches how many tons of hewn timber?

OPERATION.

```
               5|0
1728)1296000(75|0
     12096   ----
     -----        15 Tons, the Answer.
      8640
      8640
      ----
        00
```

PROOF.

```
     15
     50
    ---
    750
   1728
   ----
   6000
  1500
 5250
 750
-------
1296000 Inches.
```

2. In 5529600 solid inches, how many cords of wood? *Ans.* 25.

3. How many solid inches in a cord? *Ans.* 221184.

8. *DRY MEASURE*

1. In 75 bushels of corn how many pints?

OPERATION.

```
   75
    4
 ----
  300
    8
 ----
 2400
    2
 ----
```

Ans. 4800 *pts.*

PROOF.

```
2)4800
 ------
8)2400
 ------
 4)300
 ------
   75 Bushels.
```

2. In 9376 quarts how many bushels? *Ans.* 293.

It would be needless to give examples of Reduction in all the weights and measures. The understanding which the attentive Scholar must already have acquired of this rule, by the help of the tables, will ever be sufficient for his purpose.

SUPPLEMENT TO REDUCTION.

QUESTIONS.

1. What is Reduction?
2. Of how many kinds is Reduction? What are they called? Wherein do these kinds differ one from the other? Which of the fundamental rules are employed in their operation?
3. How is Reduction Descending performed?
4. How is Reduction Ascending performed?
5. When it is required to know how many sorts of coin, weights or measures of different values, of each an equal number, are contained in any other number of another kind, what is the method of procedure?

EXERCISES.

1. In 36 guineas, how many crowns?
Ans. 153 *crowns* & 9*d. over.*

2. How many rings, each weighing 5*pwts.* 7*grs.* may be made of 3*lb.* 5*oz.* 16*pwts.* 2*grs.* of gold?
Ans. 158

3. How many steps of 2 feet 5 inches each, will it require a man to take, travelling from Leominster to Boston, it being 43 miles?

Ans. 93947

4. Let 70 dollars be distributed among three men in such manner that as often as the first has 5*s.* the second shall have 7*s.* and the third 9*s.* What will each one receive?

Ans. first $16 4*s.* second $23 2*s.* third $30.

5. How many square feet in a square mile? *Ans.* 27878400.

6. If a vintner be desirous to draw off a pipe of Canary into bottles, containing pints, quarts, and 2 quarts, of each an equal number, how many must he have? *Ans.* 144 *of each.*

7. There are three fields, one contains 7 acres, another 10 acres, and the other 12 acres and 1 rood: how many shares of 76 perches each, are contained in the whole? *Ans.* 61 *shares and* 44 *perches over.*

8. There are 106*lb.* of silver, the property of 3 men; of which A receives 17*lb.* 10*oz.* 19*pwts.* 19*grs.* Of what remains, B shares 1*oz.* 7*grs.* so often as C shares 13*pwts.* What are the shares of B and C?

Ans. B's share 53lb. 8oz. 5pwts. 5grs. C's share 34lb. 4oz. 15pwts

§ 2. FRACTIONS.

WHEN the thing or things signified by figures are *whole ones*, then the figures which signify them are called *integers* or *whole numbers*. But when only some parts of a thing are signified by figures, as *two thirds* of any thing, *five sixths*, *seven tenths*, *&c.* then the figures which signify *these parts* of a thing being the expression of some quantity *less than one*, are called FRACTIONS.

Fractions are of two kinds, *Vulgar* and *Decimal;* they are distinguished by the manner of representing them; they also differ in their modes of operation.

VULGAR FRACTIONS.

To understand Vulgar Fractions, the learner must suppose an integer (or the number 1) divided into a number of equal parts; then any number of these parts being taken would make a fraction, which would be represented by two numbers placed one directly over the other with a short line between them thus, $\frac{2}{3}$ *two thirds*, $\frac{3}{5}$ *three fifths*, $\frac{7}{8}$ *seven eighths*, *&c.*

Each of these figures have a different name and a different signification. The figure below the line is called the denominator, and shews into how many parts an integer, or one individual of any thing is divided—the figure above the line is called the numerator, and shews how many of those parts are signified by the fraction.

For illustration, suppose a silver plate to be divided into *nine equal parts*. Now one or more of these parts make a fraction which will be represented by the figure 9 for a denominator placed underneath a short line shewing the plate to be divided into *nine equal parts;* and supposing *two* of those parts to be taken for the fraction, then the figure 2 must be placed directly above the 9 and over the line ($\frac{2}{9}$) for a numerator, shewing that two of those parts are signified by the fraction, or *two ninths* of the plate. Now let 5 parts of this plate, which is divided into 9 parts be given to John, his fraction would be $\frac{5}{9}$ *five ninths;* let 3 other parts be given to Harry, his fraction would be $\frac{3}{9}$ *three ninths;* there would then be one part of the plate remaining still (5 and 3 are 8) and this fraction would be expressed thus $\frac{1}{9}$ *one ninth.*

In this way all vulgar fractions are written; the denominator or number below the line, shewing into how many parts any thing is divided, and the numerator, or number above the line, shewing how many of those parts are taken or signified by the fraction.

To ascertain whether the learner understands what has now been taught him of fractions, let us again suppose a dollar to be cut into 13 equal parts;—let 2 of those parts be given to A; 4 to B; and 7 to C.

Required of the learner that he should write $\left\{\begin{array}{l}\text{A's fraction —}\\ \text{B's fraction —}\\ \text{C's fraction —}\end{array}\right.$

It is from division only that fractions arise in Arithmetical operations: the remainder after division is a portion of the Dividend undivided; and is always the numerator to a fraction of which the Divisor is the Denominator. The quotient is so many integers.

The Arithmetic of Vulgar Fractions is tedious and even intricate to beginners. Besides they are not of necessary use. We shall not therefore enter into any further consideration of them here. This difficulty arises chiefly from the variety of denominators, for when numbers are divided

into different kinds, or parts, they cannot be easily compared. This consideration gave rise to the invention of

DECIMAL FRACTIONS.

Decimal Fractions are also expressions of parts of an integer; or are in value something less than *one* of any thing, whatever it may be which is signified by them.

In decimals an integer, or the number *one*, as 1 foot, 1 dollar, 1 year, &c. is conceived to be divided into *ten* equal parts, (*in vulgar fractions, an integer may be divided into any number of parts*) and each of these parts is subdivided into *ten* lesser parts, and so on. In this way the denominator to a decimal fraction in all cases, will be either 10, 100, 1000, or unity (1) with a number of cyphers annexed; and this number of cyphers will always be equal to the number of places in the numerator. Thus, $\frac{6}{10}$ $\frac{27}{100}$ $\frac{685}{1000}$ are *Decimal Fractions*, of which the cyphers in the denominator of each are equal to the number of places in its own numerator.

" As the denominator of a decimal fraction is always 10, 100, 1000, &c. " the denominators need not be expressed; for the numerator only may be " made to express the true value; for this purpose it is only required to " write the numerator with a point (,) before it, called a *separatrix*, at the " left hand, to distinguish it from a whole number; thus, $\frac{6}{10}$ is written ,6; " $\frac{27}{100}$,27; $\frac{685}{1000}$,685, &c."

When integers and decimals are expressed together in the same sum, that sum is called a *mixed* number; thus, 25,63 is a mixed number; 25, or all the figures to the left hand of the separatrix being integers, and ,63 or all the figures to the right hand of the same point being decimals.

The first figure on the right hand of the decimal point signifies tenth parts; the next, hundredth parts; the next, thousandth parts, and so on.

,7	seven	signifies seven tenth parts.
,07	—	seven hundredth parts.
,27	—	two tenth parts and seven hundredth parts; or twenty seven hundredths.
,357	—	three tenth parts, five hundredth parts, and seven thousandth parts; or 357 thousandths.
5,7	—	five, and seven tenth parts.
5,007	—	five and seven thousandths.

The value of each figure from unity, and the decrease of decimals toward the right hand may be seen in the following

TABLE.

C Millions	X Millions	Millions	C Thousands	X Thousands	Thousands	Hundreds	Tens	Units	Tenth parts	Hundredth parts	Thousandth parts	X Thousandth parts	C Thousandth parts	Millionth parts	X Millionth parts	C Millionth parts
9	8	7	6	5	4	3	2	1,	2	3	4	5	6	7	8	9

Cyphers placed to the right hand of decimals do not alter their value.— Placed at the left hand they diminish their value in a tenfold proportion.

ADDITION OF DECIMALS.

RULE.

" 1. PLACE the numbers whether mixed or pure decimals, under each other according to the value of their places."

" 2. Find their sum as in whole numbers, and point off so many places " for decimals as are equal to the greatest number of decimal places in any " of the given numbers."

EXAMPLES.

1. What is the amount of 73,612 guineas, 436 guineas, 3,27 guineas, ,8632 of a guinea, and 100,19 guineas when added together.

OPERATION.

```
 73,612
436,
  3,27
   ,8632
100,19
--------
```

Ans. 613,9352 *guineas.*

The decimals are arranged from the separatrix towards the right hand, and the whole numbers from the same point towards the left hand. The greatest number of decimal places in any of the numbers is four, consequently four figures in the product must be pointed off for decimals.

2.

```
345,601
   ,3724
 63,1
572,313
  7,5462
--------
```

3. Required the sum of 37,821+546,35+8,4+37,325.

Ans. 629,896.

4. What is the sum of three hundred twenty-nine and seven tenths; thirty-seven and one hundred and sixty-two thousandths; and sixteen hundredths, when added together?

Ans. 367,022.

5. Add six hundred and five thousandths, and four thousandth and three hundredths?

Sum 4600,035.

Note.—When the numerator has not so many places as the denominator has cyphers, prefix so many cyphers at the left hand as will make up the defect; so $\frac{5}{1000}$ is written thus, ,005, &c.

SUBTRACTION OF DECIMALS.

RULE.

" Place the numbers according to their value; then subtract as in whole numbers, and point off the decimals as in Addition."

EXAMPLES.

1. From 716,325 take 81,6201.

OPERATION.

From	716,325
Take	81,6201
	634,7049

2. From 119,1384 take 95,91.
Rem. 23,2284.

3. What is the difference between 287 and 3,115? *Ans.* 283,885.

4. From 67, take ,92.
Rem. 66, 08.

All the operations in Decimal Fractions are extremely easy; the only liability to error will be in placing the numbers and pointing off the decimals; and here care will always be security against mistakes.

MULTIPLICATION OF DECIMALS.

RULE.

" 1. Whether they are mixed numbers or pure decimals, place the factors, and multiply them as in whole numbers."

" 2. Point off so many figures from the product as there are decimal places in both the factors; and if there be not so many decimal places in the product, supply the defect by prefixing cyphers."

EXAMPLES.

1. Multiply ,0261 by ,0035.

OPERATION.

```
  ,0261
  ,0035
  -----
   1305
   783
  -----
,00009135 Product.
```

In this example, the decimals in the two factors taken together are *eight;* the product falls short of this number by *four* figures, consequently, four cyphers are prefixed to the left hand of the product.

2. Multiply 31,72 by 65,3.
Product, 2071,316.

OPERATION
3 1,7 2
6 5,3

3. **Multiply 25,238 by 12,17**
***Product*, 307,14646.**

4. **Multiply ,62 by ,04.**
***Product*, ,0248.**

5. **Multiply 17,6 by ,75.**
***Product*, 13,2.**

DIVISION OF DECIMALS.

RULE.

"1. The places of decimal parts in the divisor and quotient counted together must be always equal to those in the dividend, therefore divide as in whole numbers, and from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

"2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

"3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend or to the remainder, and the quotient carried on to any degree of exactness."

EXAMPLES.

Divide 2,735 by 51,2.

OPERATION.

51,2)2,735(,0534+
2,560
———
1750
1536
———
2140
2048
———
92

In this example there are *five decimals* in the dividend (counting the two cyphers which were added to the remainder of the dividend after the first division) that the decimals in the divisor and quotient counted together may equal that number, a cypher is prefixed to the left hand of the quotient.

In the division of decimals it is proper to add cyphers so long as there continues to be a remainder, this however is not practised, nor is it necessary; four or five decimals being sufficiently accurate for most calculations.

2. Divide 3156,293 by 25,17.
Quotient, 1253+

NOTE. The separatrix is omitted in the answers to the examples on this page to exercise the scholar in placing it according to rule; to this the Instructor should be particularly attentive.

3. Divide 5737 by 13,8.
Quotient, 431353+

4. Divide 173948 by ,375.
Quotient, 463861+

5. Divide 2 by 53,1
Quotient, 037+

6. Divide ,012 by ,005.
Quotient, 24.

REDUCTION OF DECIMALS.

CASE 1.

TO REDUCE VULGAR FRACTIONS TO DECIMALS.

RULE.

ANNEX a cypher to the numerator and divide it by the denominator, annexing a cypher continually to the remainder. The quotient will be the decimal required.

EXAMPLES.

1. Reduce $\frac{3}{5}$ to a decimal.

OPERATION.

```
5)3,0(,6 Ans.
  3 0
  ---
  0 0
```

The numerator in these operations is considered as an integer, and always requires the decimal point to be placed immediately after it, the cyphers annexed occupy the places of decimals, the quotient must be pointed off according to the rule in division.

2. Reduce $\frac{1}{7}$ to a decimal

OPERATION.

```
7)1,0(,1428+Ans.
  7
  --
  30
  28
  --
   20
   14
   --
    60
    56
    --
     4
```

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ to decimals. *Answers*, ,25 ,5. ,75.

4. Reduce $\frac{5}{26}$, $\frac{12}{480}$, and $\frac{9}{1129}$ to decimals. *Ans.* ,1923+,025 ,00797+

CASE 2.

To reduce numbers of different denominations, as of Money, Weight and Measure to their decimal values.

RULE.

"I. Write the given numbers perpendicularly under each other for
" dividends, proceeding orderly from the least to the greatest.

"II. Opposite to each dividend on the left hand, place such a number "for a divisor as will bring it to the next superior denomination and draw "a line perpendicularly between them.

"III. Begin with the highest and write the quotient of each division, as "decimal parts on the right hand of the dividend next below it, and so on, "till they are all used, and the last quotient will be the decimal sought."

EXAMPLES.

1 Reduce 10*s*. 6¾*d*. to the fraction of a pound.

OPERATION.

4	3,
12	6,75
20	10,5625
	,528125 *Ans.*

The given numbers arranged for the operation, all stand as integers. I then suppose 2 cyphers annexed to the 3(3,00) which divided by 4, the quotient is 75, which I write against six in the next line, and the sum thus produced (6,75) I divide by 12, placing the quotient, (5625) at the right hand of the 10; lastly, I divide by 20 and the quotient (,528125) is the decimal required.

2. Reduce 13*s*. 5½*d*. to the decimal of a pound. *Ans.* ,6729+

3. Reduce 12*pwts.* 14*grs.* to the decimal of an ounce. *Ans.* ,6291.

CASE 3

To find the value of any given decimal in the terms of an integer.

RULE.

Multiply the decimal by that number which it takes of the next less denomination to make *one* of that denomination in which the decimal is given, and cut off so many figures for a remainder to the right hand of the quotient, as there are places in the given decimal. Proceed in the same manner with the remainder, and continue to do so through all the parts of the integer, and the several denominations standing on the left hand make the answer.

EXAMPLES.

1. What is the value of ,528125 of a pound?

	OPERATION.
	,5 2 8 1 2 5
	2 0
Shillings 1	0,5 6 2 5 0 0
	1 2
Pence	6,7 5 0 0 0 0
	4
Farthings	3,0 0 0 0 0 0

Ans. 10*s.* 6$\frac{3}{4}$*d.*

This question is the first example in the preceding case inverted, by which it will be seen that questions in these two cases may reciprocally prove each other.

The given decimal being the decimal of a pound, and shillings being the next less inferior denomination, because 20 shillings make one pound, I multiply the decimal by 20, and cutting off from the right hand of the product a number of figures, for a remainder equal to the number of figures in the given decimal, leaves 10 on the left hand which are shillings. I then multiply the remainder, which is the decimal of a shilling by 12, and cutting off as before, gives 6 on the left hand for pence; lastly, I multiply this last remainder, or decimal of a penny by 4, and find it to be 3 farthings, without any remainder. It then appears that ,528125 of a pound is in value 10*s.* 6$\frac{3}{4}$*d.*

2. What is the value of ,73968 of a pound? *Ans.* 14*s.* 9$\frac{1}{2}$*d.*

3. What is the value of ,768 of a pound Troy?
Ans. 9*oz.* 4*pwt.* 7$\frac{17}{25}$**grs.*

* $\frac{17}{25}$ is the last remainder, 680 reduced to its lowest terms. A fraction is said to be reduced to its lowest terms, when there is no number which will divide both the numerator and denominator without a remainder.—Thus, set to the fraction its proper denominator $\frac{680}{1000}$, then divide the numerator and the denominator by any number which will divide them both, without a remainder, continue to do so as long as any number can be found that will divide them in that manner.

4.

$$8)\frac{680}{1000}=\frac{85}{125}=\frac{17}{25}.$$

SUPPLEMENT TO FRACTIONS.

QUESTIONS.

1. What are fractions?
2. What are integers or whole numbers?
3. What are mixed numbers?
4. Of how many kinds are fractions?
5. How are Vulgar Fractions written?
6. What is signified by the denominator of a fraction?
7. What is signified by the numerator?
8. How are Decimal Fractions written?
9. How do Decimals differ from Vulgar Fractions?
10. How can it be ascertained what the denominator to a Decimal Fraction is if it be not expressed?
11. How do cyphers placed at the left hand of a Decimal Fraction affect its value?
12. How are Decimals distinguished from whole numbers?
13. In the addition of Decimals what is the rule for pointing off?
14. What is the rule of pointing off Decimals in Subtraction? In Multiplication? and in Division?
15. In what manner is the reduction of a Vulgar Fraction to a decimal performed?
16. How are numbers of different denominations, as pounds, shillings, pence, &c. reduced to their decimal values?
17. If it be required to find the value of any given decimal in the terms of an integer, what is the method of procedure?

EXERCISES.

1. What is the sum of $79\frac{1}{2}$ $6\frac{1}{4}$ and of $\frac{3}{4}$ when added together.

OPERATION.

```
79,5
 6,25
  ,75
------
86,50 Ans.
```

2. From 17 take $\frac{3}{4}$.

OPERATION.

```
17,
  75
------
16,25 Remainder.
```

In Case 1. *Ex.* 3d, under Reduction of decimal fractions, the Scholar may notice that $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ reduced to decimals are, ,25, ,5 and ,75. When numbers, therefore, for operations in either of the fundamental Rules, are incumbered with these fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, substitute for them

3. Multiply $68\frac{1}{4}$ by $5\frac{1}{2}$.

OPERATION.

```
      6 8, 2 5
          5, 5
    ----------
    3 4 1 2 5
  3 4 1 2 5
  ------------
  3 7 5, 3 7 5 Product.
```

4. Divide $26\frac{1}{4}$ by $2\frac{1}{2}$.

OPERATION.

```
2,5)26,25(10,5 Quotient.
    25
    ---
     125
     125
    ---
```

their equivalent decimal fractions, that is, for $\frac{1}{4}$,25 for $\frac{1}{2}$,5 for $\frac{3}{4}$,75 then proceed according to the rules already given for these respective operations in decimal fractions.

Many persons are perplexed by occurrences of a similar nature to the examples above. Hence is seen in some measure the usefulness of fractions, particularly decimal fractions. The only thing necessary to render any person adroit in these operations is to have riveted in his mind the rules for pointing as taught and explained in their proper places. They are not burthensome; every scholar should have them perfectly committed.

5. If a pile of wood be 18 feet long, $11\frac{1}{4}$ wide, and $7\frac{3}{4}$ high, how many cords does it contain?

Ans. 12 *cords*, 68 *feet*,* 432 *inches*.

A cord of wood is 128 solid feet; the proportions commonly assigned are, 8 feet in length, 4 in breadth, and 4 in height.

The contents of a load or pile of wood of any dimensions may be found by multiplying the length by the breadth, and this product by the height; or, by multiplying the length, breadth and height into each other. The last product divided by 128 will shew the number of cords, the remainder, if any, will be so many solid feet.

* The 432 inches is the fraction, ,25 of a foot, valued according to CASE 3, Reduction Decimal Fractions.

6. If a load of wood be 9 feet long, $3\frac{1}{2}$ feet wide, and 4 feet high, how many square feet does it contain?
Ans. 126 *feet, which are two feet short of a cord.*

7. What is the value of ,725 of a day? *Ans.* 17 *hrs.* 24 *min.*

8. What is the value of ,0625 of a shilling? *Ans.* 3 *farthings.*

9. Reduce 3*Cwt.* 0*qrs.* 7*lb.* 8*oz.* to the decimal of a ton.
Ans. ,15334821+

10. Reduce 3 farthings to the decimal of a shilling? *Ans.* ,0625.

11 Reduce $\frac{2}{160}$ to a decimal fraction. *An*. 0125.

§ 3. FEDERAL MONEY.

Federal Money is the coin of the United States, established by Congress, A. D. 1786. Of all coins this is the most simple, and the operations in it the most easy.

The denominations are in a *decimal proportion*, as exhibited in the following

TABLE.

10 Mills	make one	Cent,
10 Cents	make one	Dime,
10 Dimes	make one	Dollar, *marked thus*, $
10 Dollars	make one	Eagle.

The expression of any sum in Federal Money is simply the expression of a *mixed number* in decimal fractions. A dollar is the *Unit Money;* dollars therefore must occupy the place of units, the less denominations, as dimes, cents, and mills, are decimal parts of a dollar, and may be distinguished from dollars in the same way as any other decimals by a comma or separatrix. All the figures to the left hand of dollars, or beyond units place are eagles. Thus, 17 eagles, 5 dollars, 3 dimes, 4 cents, and 6 mills are written—

Eagles; or, Hundreds.	Tens.	Doll's; or, Units.	Dimes; or, Tenth parts.	Cents; or, Hundredth parts.	Mills; or, Thousandth parts.
1	7	5,	3	4	6

Of these, four are real coins, and one is imaginary. The real coins are the Eagle, a gold coin; the Dollar and the Dime, silver coins; and the Cent, a copper coin. The Mill is only imaginary, there being no piece of money of that denomination.

There are half eagles, half dollars, double dimes, half dimes, and half cents, real coins.

These denominations, or different pieces of money, being in a tenfold proportion, consequently any sum in Federal Money does of itself exhibit the particular number of each different piece of money contained in it. Thus, 175,346 (*seventeen eagles, five dollars, three dimes, four cents, six mills*) contain 175346 mills, 17534 $\frac{6}{10}$ cents, 1753 $\frac{46}{100}$ dimes, 175 $\frac{346}{1000}$ dolls. 17 $\frac{5346}{10000}$ eagles. Therefore, eagles and dollars reckoned together, express the number of dollars contained in the sum; the same of dimes and cents; and this indeed is the usual way of account, to reckon the whole sum in dollars, cents, and mills, thus:

Dolls.	*Cents.*	*Mills.*
$175	34	6

The Addition, Subtraction, Multiplication and Division of Federal Money is performed in all respects as in Decimal Fractions, to which the Scholar is referred for the use of rules in these operations.

ADDITION OF FEDERAL MONEY.

1. Add 16 Eagles; 3 Eagles, 7 Dollars 5 Cents; 26 Dollars, 6 Dimes, 4 Cents, 3 Mills; 75 Cents, 8 Mills, 40 Dollars, 9 Cents together.

OPERATION.

Eag.	*Dolls.*	*Dimes.*	*Cents.*	*Mills.*
1 6	0,			
3	7,	0	5	
2	6,	6	4	3
	,	7	5	8
4	0,	0	9	
$2 6	4,	5	4	1

2. If I am indebted 59 dollars, 112 dollars, 98 cents, 113 dolls. 15 cts. 15 dollars, 21 dollars, 50 cents, 200 dollars, 73 dollars, 35 dollars, 17 cents, 75 dollars, 20 dollars, 40 dollars, 33 cents and 16 dollars. What is the sum which I owe? *Ans.* $781 13.

Or the sums may be all reckoned in dollars, cents and mills, thus,

Dolls.	*Cents.*	*Mills.*
$160		
37	05	
26	64	3
	75	8
40	09	
$264	54	1

Accountants generally omit the comma, and distinguish *cents* from *dollars* by setting them apart from the dollars.

SUBTRACTION OF FEDERAL MONEY.

1. From $863, 17 take $69, 82.

OPERATION.

	8 6 3, 1 7
	6 9, 8 2
Remainder,	7 9 3, 3 5

2. From $681 take $57,63, *Remainder,* $623,37.

MULTIPLICATION OF FEDERAL MONEY.

1. If flour be $10,25 per barrel, what will 27 barrels cost?

OPERATION.

```
  1 0, 2 5
      2 7
  -------
  7 1 7 5
2 0 5 0
---------
$2 7 6, 7 5 Ans.
```

Point off the decimals in the product according to the rule in Multiplication of decimals; if at any time there shall be more than three decimal figures, all beyond mills or the third place, will be decimal parts of a mill.

2. Multiply $76,35 by $37,46.
Product, $2860,0710.

3. Multiply $24,675 by $13,63.
Product, $336,320$\frac{25}{100}$.

DIVISION OF FEDERAL MONEY.

1. If 2728 bushels of wheat cost $2961, how much is it per bushel?

OPERATION.

```
Bushels. Dolls. D. d. c. m.
   2728)2961(1, 0 8 5 Ans.
        2728
        ----
         23300
         21824
         -----
          14760
          13640
          -----
           1120
```

When the dividend consists of dollars only, if there be a remainder after division, cyphers must be annexed as in division of decimals.

2. Divide $3756 equally among 13 men; what will each man receive? *Ans.* $288,923.

3. Divide $16,75 by 27.
Quotient, 62 cents

SUPPLEMENT TO FEDERAL MONEY.

QUESTIONS.

1. What is FEDERAL MONEY? When was its establishment, and by what authority?
2. What are the denominations in Federal Money?
3. Which is the Unit Money?
4. How are dollars distinguished from dimes, cents, and mills?
5. What places do the different denominations occupy, from the decimal point?
6. How is the addition of Federal Money performed? Subtraction? Multiplication? Division?

EXERCISES.

1. A man dies, leaving an estate of $71600, there are demands against the estate of $39876,74; the residue is to be divided between 7 sons; what will each one receive?
Ans. $4531 89*cts.*

2. A man sells 1225 bushels of wheat at $1,33 per bushel, and receives $93,76 for transportation; what does he receive in the whole?
Ans. $1723,01.

3. What will 3 hogsheads of sugar cost, each weighing 3*Cwt.* 2*qrs.* 7*lb.* at 16*cts.* 7*mills* per *lb.*?
Ans. $199,899.

4. Divide seven thousand six dollars, one cent and three mills, by five hundred seventy six dollars, thirty four cents and two mills.
Ans. $12,155

§ 4. EXCHANGE.

EXCHANGE is the giving of the bills, money, weight, or measure of one place or country, for the like value in the bills, money, weight or measure of another place or country.

NOTE 1. The Currencies in the New England States, and in Virginia, are the same and will be all comprehended under the term *N. E. currency;* those of New-York, North Carolina and Ohio, are the same, and will be comprehended under the term *N. York Currency;* those of N. Jersey, Pennsylvania, Delaware and Maryland, are the same, and will all be comprehended under the term *Penn. Currency.*

NOTE 2. It will be sufficient perhaps in most cases, that the pupil be required to work this rule in the currency of that State only to which he belongs.

CASE 1.

To change New-England, &c. and New-York, &c. Currencies to Federal Money.

RULE.

Set down the pounds, and to the right hand write half the greatest even number of the given shillings: then consider how many farthings there are contained in the given pence and farthings, and if the sum exceed 12, increase it by 1, or if it exceed 36, increase it by 2, which sum set down to the right hand of half the greatest even number of shillings before written, remembering to increase the second place, or the place next to shillings by 5, if the shillings be an odd number; to the whole sum thus produced, annex a cypher, and divide the sum by 3, if it be *N. England* currency, and by 4 if it be *New-York;* cut off the three right hand figures in the quotient, which will be cents and mills; the rest will be dollars.

EXAMPLES.

1. Change £47 7*s.* 10¾*d.* to dollars, cents and mills.

OPERATION.

	The pounds.	Half the even number of shillings.	The farthings in pence and farthings increased according to rule.	Cypher annexed.
Divide by 3)	4 7	3	9 5	0
Dolls.	1 5	7,	9 8	3

In this example to the right hand of pounds (47) I write 3, half the greatest even number of the given shillings(7); the farthings in 10¾*d.* (43) increased by 2 (45) because exceeding 36 and the second place increased by 5 because the shillings were an odd number, make 95, which sum written to the right hand of the 3, a cypher annexed, and the sum divided by 3 gives the answer 157 *dollars*, 98 *cents*, and 3 *mills* for *N. England currency;* the same sum (473950) divided by 4, gives 118 *dollars*, 48 *cents*, 7 *mills* for *N. York currency.*

If there be no shillings, or only 1 *shilling in the given sum, so there be no even number,* write a cypher in place of half the even number of shillings, then proceed with the pence and farthings as in other cases.

If pounds only are given to be changed, annex a cypher and divide as before, the quotient will be dollars. If there be a remainder, annex 3 more cyphers and divide, the quotient will be cents and mills.

If pounds and an even number of shillings only be given, to the pounds annex half the even number of shillings, divide as before, and the quotient will be dollars

A little practice will make these operations extremely easy.

2. In £763 *N. E.* and *N. Y.* currencies, how many dollars, cents, and mills?

Ans. $2543 33*cts.* 3*m.* *N. E. cur.*
1907 50 *N. Y.* —

3. In £17 1*s.* 6½*d.* how many dollars, cents and mills?

Ans. $56 92 3. *N. E. cur.*
42 69 2. *N. Y.* —

4. In £109 3*s.* 8*d.* how many dollars and cents?

Ans. $363,94 *N. E. cur.*
272,95 *N. Y.* —

5. In £86 6*s.* 5¼*d.* how many dollars, cents and mills?

Ans. $287,740 *N. E. cur.*
215,805 *N. Y.* —

6. Exchange £1 1*s.* 10¼*d.* to Federal Money.

Ans. $3,646 *N. E. cur.*
2,735 *N. Y.* —

7. Exchange £10 4¼*d.* to Federal Money.

Ans. $33,396 *N. E. cur.*
25,047 *N. Y.* —

8. Exchange £103 to Federal Money.

Ans. $343,333 *N. E. cur.*
257,50 *N. Y* -

9. Exchange 2¾*d.* to Federal Money.

Ans. 3*cts.* 6*m.* *N. E. cur*
2— 7— *N. Y.* —

CASE 2.

To Exchange Federal Money to New-England and New-York Currencies.

RULE.

If there be no mills in the given sum, reduce it to mills by annexing cyphers; multiply the given sum by 3, if it be required to change it to N. E. currency; but if to the currency of N. York, by 4; cut off the four right hand figures, which will be decimals of a pound, the left hand figures will be the pounds. To find the value of the decimals, double the first figure for shillings, and if the figure in the second place be 5, add another shilling, then call the figures in the second and third places, after deducting the 5 in the second place, so many farthings, abating 1 when they are above 12, and 2, when they are above 36.

EXAMPLES.

1. Change 255 dollars, 40 cents, 6 mills, to pounds, shillings, pence and farthings.

OPERATION.	OPERATION.
2 5 5 4 0 6	2 5 5 4 0 6
3	4
7 6|6 2 1 8	1 0 2|1 6 2 4
Ans. £76 12*s.* 5*d.* *N. E. cur.*	£102 3*s.* 3*d.* *N. Y. cur.*

Having multiplied and cut off the four right hand figures as the rule directs, to find the value of the figures cut off, I double the first figure (6) *N. E cur.* which gives 12 for shillings; the figures in the second and third places (21) abating 1 for being over twelve (20) are to be considered so many farthings, which reduced to pence are 5.

The 3*s.* 3*d. N. Y. cur.* are obtained after the same manner. The double of the first figure cut off (1) is 2, and because the figure in the second place (6) is more than 5, I add another shilling, making 3*s.* then the figures in the second and third places (62) after deducting the 5 for 1 shilling from the 6, are 12, which reduced to pence are 3.

The 8 and the 4 in the fourth places, being something less than one farthing, are lost, not being reckoned.

If there be neither cents nor mills, that is, *if the given sum be dollars*, multiply by 3 and cut off one figure only.

2. In $392,75 how many pounds, shillings, pence and farthings?

Ans. £117 16*s.* 6*d.* *N. E. cur.*
157 2 0 *N. Y.* —

3. In $39,635 how many pounds, shillings, pence, &c.?

Ans. £11 17*s.* 9½*d.* *N. E. cur.*
15 17 1 *N. Y. cur.*

4. Exchange 134 dollars 65 cents to pounds, shillings, pence and farthings.

Ans. £40 7*s.* 10¾*d.* *N. E. cur.*
53 17 2½ *N. Y. cur.*

5. Exchange 684 dollars to pounds and shillings.

Ans. £205 4*s.* *N. E. cur.*
273 12 *N. Y. cur.*

6. Exchange 71 cents to shillings, pence, &c.

Ans. 4*s.* 3*d.* *N. E. cur.*
5 8¼ *N. Y. cur.*

7. Exchange 13*cts.* 7*m.* to pence and farthings.

Ans. 9¾*d.* *N. E. cur.*
13*d.* *N. Y. cur.*

CASE 3.

To change New-Jersey, Pennsylvania, Delaware and Maryland Currency to Federal Money.

RULE.

Reduce the given sum to pence, annex a cypher, divide these pence by 9, and add the quotient to the pence; from the sum point off three figures, which will be cents and mills; those to the left hand will be dollars.

If there are farthings in the given sum, in place of the cypher annex 2 for 1 farthing; 5 for 2 farthings; 7 for 3 farthings, and proceed as before.

If the given sum be pounds only, multiply by 8, annex 3 cyphers to the product, and divide by 3; the quotient will be the answer, pointing off the three right hand figures for cents and mills.

EXAMPLES.

1 Change £17 1*s.* 6½*d.* to Federal Money.

```
      20
    ----
     341
      12
   -----
 9)40985
    4553*
   -----
Ans. 45,538
```

I first reduce the given sum to pence, to which (4098) I annex the figure 5 for the ½*d.* and divide by 9; the quotient added to the pence and the three right hand figures pointed off give the answer, 45 dollars, 53 cents and 8 mills.

* This quotient figure (3) might with propriety have been put down 4, the 9's in 35 coming so near producing it, and it would have been nearer the true value; the mills in the answer would then have been 9 in place of 8.

2. In £109 3*s.* 8*d.* how many dollars, cents and mills?
Ans. $291,155.

3. Change £736 to Federal Money.
Ans. $1962,666.

4. In £86 6*s.* 5¼*d.* how many dollars, cents and mills?
Ans. $230,191.

5. Change 6¾*d.* to Federal Money
Ans. 7*cts.* 4 *mills.*

CASE 4.

To change Federal Money to New-Jersey, Pennsylvania, Delaware and Maryland Currency.

RULE.

If there be no mills in the given sum, reduce it to mills by annexing cyphers, subtract one tenth of itself, the remainder, except the rignt hand figure, will be pence, which must be reduced to pounds; to find the value of the right hand figure, if it be 2, reckon 1 farthing; if 5, reckon it 2 farthings; if 7, reckon it 3 farthings.

NOTE.—Subtracting the tenth of the given sum from itself may be done in this manner:—Suppose the sum 6452. Write the given sum under itself, removing the figures one place towards the right hand and dropping the right hand figure; subtract and the remainder will be the sum required.

```
6 4 5 2
  6 4 5
-------
5 8 0 7
```

EXAMPLES.

1. Change $45,538 to pounds, shillings, pence and farthings.

OPERATION.

```
   45,538
    4,553
 --------
12)4098|5
 --------
  2|0)34|1
Ans. £17 1s. 6½d.
```

This is the first example in the former Case inverted. Having subtracted one tenth of the given sum from itself in the manner directed in the note above, the right hand figure in the remainder (5) being to be reckoned 2 farthings, I set it down in the answer ½*d.*—the other figures of the remainder (4098) being pence, I divide by 12, in doing which there is a remainder of 6, which are pence; these I also set down in the answer. The shillings (341) divided by 20, cutting off one figure from the divisor and one from the dividend as is usually practised in reducing shillings to pounds, give £17, and the 1 cut off from the dividend is 1 shilling, which completes the answer.

2. Change $135 to pounds, &c.
Ans. £50 12*s.* 6*d.*

3. Change $287,74 to pounds
Ans. £107 18*s.* 0½*d.*

To change the New-England to the New-York currency; add one third.

To change the New-York to the New-England currency; subtract one fourth.

To change the New-England to the Pennsylvania currency; add one fourth.

To change the Pennsylvania to the New-England currency; subtract one fifth.

To change the New-York to the Pennsylvania currency; subtract one sixteenth.

To change the Pennsylvania to the New-York currency? add one fifteenth.

SUPPLEMENT TO EXCHANGE

QUESTIONS.

1. *What is Exchange?*

2. How do you change N. England and Virginia currencies to Federal Money?—New-York currency?—and wherein consists the difference?

3. If pounds only are given to be changed, how do you proceed?

4. When there are no shillings, or only one in the given sum, how do you proceed?

5. How do you change Federal Money to N. England currency? N. York currency?—Wherein consists the difference?

2. How do you change Pennsylvania, &c. currency to Federal Money?

3. If there are farthings in the given sum, how do you proceed?

4. If the given sum be pounds only, how do you proceed?

5. How do you change Federal Money to Pennsylvania, &c. currency?

6. How do you change New-England to New-York currency?—New York to New-England?—New-England to Pennsylvania?—Pennsylvania to New-England?—New-York to Pennsylvania?—Pennsylvania to New-York currency?

EXERCISES.

1. In £36 1*s*. 6½*d*. N. Eng. *cur*. or £48 2*s*. 0½*d*. N. York *cur*. or £45 1*s*. 11*d*. Penn. *cur*. how many dollars, cents and mills?
Ans. $120,257 N. E. *cur*.—$120,255 N. Y. *cur*.—$120, 255 Penn. *cur*.

Note.—In making the exchange from one currency into another there will frequently be the loss of some fractions of a farthing; for this reason when the exchange is again made into Federal Money, there will be the difference of some mills in the answers obtained.

2. Change £180 12*s*. N. E. *cur*. to N. Y. *cur*. Penn. *cur*. and to Federal Money.
Ans. £240 16*s*. N. Y. *cur*.—£225 15*s*. Penn. *cur*.—$602 *F. Money*.

3. Change $150,25 to N. England, N. York, or Penn. *cur*. accordingly as the pupil may have been instructed in one or the other, or all of these rules.
Ans. £45 1*s*. 6*d*. N. E. *cur*.—£60 2*s*. N. Y. *cur*.—£56 6*s*. 10½*d*. Penn. *cur*.

4. Let the pupil be required to change the sums in New-York and in Pennsylvania currency in the above answer, to New-England currency; the same in New-England and in New-York to Pennsylvania currency; and the same in New-England and Pennsylvania to New-York currency, the answers of which will reciprocally prove each other.

5. Change $345,625 to N. Eng. or N. York, or Penn. currency.
Ans. £103 13*s*. 8¾*d*. N. E. *cur*.—£138 5*s*. N. Y. *cur*.—£129 12*s*. 2¼*d* Penn. *currency*.

6. Change 75 cents into N. E. or N. Y. or Penn. *cur*.
Ans. 4*s*. 6*d*. N. E. *cur*.—6*s*. N. Y. *cur*.—5*s*. 7½*d*. Penn. *cur*.

7. Change £45 1*s*. 6*d*. N. E. *cur*. or £60 2*s*. N. Y. *cur*. or £56 6*s*. 10½*d* Penn. *cur*. to Federal Money. *Ans*. $150,25.

8. Change 4*s*. 6*d*. N. E. *cur* or 6*s*. N. Y. *cur*. or 5*s*. 7½*d*. Penn. currency to Federal Money. *Ans*. 75 *cents*.

9. Change £46 10*s*. 6½*d*. considered in either currency to Federal Money.
Ans. $155,09 N. E. *cur*.—$116,317 N. Y. *cur*.—$124,072 Penn. *cur*.

10. Change $167 to N. E. or N. Y. or Penn. currency.
Ans. £50 2*s*. N. E. *cur*.—£66 16*s*. N. Y. *cur*.—£62 12*s*. 6*d*. Penn. *cur*.

11. Let the pupil be required to change the sums in New-York and Pennsylvania currency, in the above answer, to New-England currency, &c. as in the 4th exercise above.

12. Change 6½*d*. to Federal Money.
Ans. 9 *cents* N. E. *cur*.—6 *cents* 7 *mills* N. Y. *cur*.—7 *cents* 2 *mills* Penn. *currency*.

13. Change £263 to Federal Money.
Ans. $876,666 N. E. *cur*.—$657,50 N. Y. *cur*.—$701,333 Penn. *cur*

TABLE

FOR REDUCING NEW-ENGLAND CURRENCY TO FEDERAL MONEY.

			shill.		shill.		shill.		shill.		shill.	
	0		1		2		3		4		5	
Pence.	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*
0			16	7	33	3	50		66	7	83	3
1	1	4	18	1	34	7	51	4	68	1	84	7
2	2	8	19	5	36	1	52	8	69	5	86	1
3	4	2	20	9	37	5	54	2	70	9	87	5
4	5	6	22	3	38	9	55	6	72	3	88	9
5	7		23	7	40	3	57		73	7	90	3
6	8	3	25		41	6	58	3	75		91	6
7	9	7	26	4	43		59	7	76	4	93	
8	11	1	27	8	44	4	61	1	77	8	94	4
9	12	5	29	2	45	8	62	5	79	2	95	8
10	13	9	30	6	47	2	63	9	80	6	97	2
11	15	3	32		48	6	65	3	82		98	6

TABLE

FOR REDUCING NEW-YORK CURRENCY TO FEDERAL MONEY.

			shill.		shill.		shill.		shill.		shill.	
	0		1		2		3		4		5	
Pence.	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*	*Cts.*	*M.*
0			12	5	25	0	37	5	50	0	62	5
1	1	0	13	5	26	0	38	5	51	0	63	5
2	2	1	14	6	27	1	39	6	52	1	64	6
3	3	1	15	6	28	1	40	6	53	1	65	6
4	4	2	16	7	29	2	41	7	54	2	66	7
5	5	2	17	7	30	2	42	7	55	2	67	7
6	6	2	18	7	31	2	43	7	56	2	68	7
7	7	2	19	7	32	2	44	7	57	2	69	7
8	8	3	20	8	33	3	45	8	58	3	70	8
9	9	3	21	8	34	3	46	8	59	3	71	8
10	10	5	23	0	35	5	48	0	60	5	73	0
11	11	5	24	0	36	5	49	0	61	5	74	0

To find by these Tables the Cents and Mills in any sum of shillings and pence under one dollar, look the shillings at top, and the pence in the left hand column ; then under the former, and on a line with the latter, will be found the cents and mills sought.

TABLE

FOR REDUCING THE CURRENCIES OF THE SEVERAL UNITED STATES TO FEDERAL MONEY.

		N. Hamp. Mass. Rh. Island. Conn. and Virginia.	New-York and N. Carolina.	N. Jersey, Pennsylva'a Delaware, and Maryland.	S. Carolina, and Georgia.
		D. cts. m.	D. cts. m.	D. cts. m.	D. cts. m.
Farthings.	1	, 3	, 3	, 3	, 4
	2	, 7	, 5	, 6	, 9
	3	, 10	, 8	, 8	, 14
Pence.	1	, 14	, 10	, 11	, 18
	2	, 28	, 21	, 22	, 36
	3	, 42	, 31	, 33	, 54
	4	, 56	, 42	, 44	, 71
	5	, 69	, 52	, 56	, 89
	6	, 83	, 62	, 67	,107
	7	, 97	, 73	, 78	,125
	8	,111	, 82	, 89	,143
	9	,125	, 94	,100	,161
	10	,139	,104	,111	,179
	11	,153	,114	,122	,196
Shillings.	1	,167	,125	,133	,214
	2	,333	,250	,267	,429
	3	,500	,375	,400	,643
	4	,666	,500	,533	,857
	5	,833	,625	,667	1,071
	6	1,000	,750	,800	1,286
	7	1,167	,875	,933	1,500
	8	1,333	1,000	1,067	1,714
	9	1,500	1,125	1,200	1,929
	10	1,667	1,250	1,333	2,143
	11	1,833	1,375	1,467	2,357
	12	2,000	1,500	1,600	2,571
	13	2,167	1,625	1,733	2,785
	14	2,333	1,750	1,867	3,000
	15	2,500	1,875	2,000	3,214
	16	2,667	2,000	2,133	3,428
	17	2,833	2,125	2,267	3,643
	18	3,000	2,250	2,400	3,857
	19	3,167	2,375	2,533	4,071

TABLE

FOR REDUCING THE CURRENCIES, &c. CONTINUED.

	New-Hamp. &c. &c.	New-York, &c.	New-Jersey, &c.	S. Carolina, &c.
£.	*D. c. m.*	*D. c. m.*	*D. c. m.*	*D. c. m.*
1	3,333	2,5	2,666	4,286
2	6,667	5,0	5,333	8,571
3	10,000	7,5	8,000	12,857
4	13,333	10,0	10,667	17,143
5	16,667	12,5	13,333	21,429
6	20,000	15,0	16,000	25,714
7	23,333	17,5	18,667	30,000
8	26,667	20,0	21,333	34,286
9	30,000	22,5	24,000	38,571
10	33,333	25,0	26,667	42,857
20	66,667	50,0	53,333	85,714
30	100,000	75,0	80,000	128,571
40	133,333	100,0	106,667	171,429
50	166,667	125,0	133,333	214,286
60	200,000	150,	160,000	257,143
70	233,333	175,	186,667	300,000
80	266,667	200,	213,333	342,857
90	300,000	225,	240,000	385,714
100	333,333	250,	266,667	428,571
200	666,667	500,	533,333	857,143
300	1000,000	750,	800,000	1285,714
400	1333,333	1000,	1066,667	1714,286
500	1666,667	1250,	1333,333	2142,857
600	2000,000	1500,	1600,000	2571,429
700	2333,333	1750,	1866,667	3000,000
800	2666,667	2000,	2133,333	3428,571
900	3000,000	2250,	2400,000	3857,143
1000	3333,333	2500,	2666,667	4285,714

TABLE

FOR REDUCING FEDERAL MONEY TO THE CURRENCIES OF THE SEVERAL UNITED STATES.

	New-Hamp. &c. &c. DOLL. 6*s.*	New-York, &c. DOLL. 8*s.*	N. Jersey, &c. &c. DOLL. 7*s.* 6*d.*	S. Carolina, &c. DOLL. 4*s.* 6*d.*
D. cts.	*£. s. d. q.*	*£. s. d. q.*	*£. s. d. q.*	*£. s. d. q.*
,01	3	1 0	1 0	2
,02	1 2	2 0	1 3	1 0
,03	2 1	3 0	2 3	1 3
,04	3 0	3 3	3 2	2 1
,05	3 2	4 3	4 2	2 3
,06	4 1	5 3	5 2	3 1
,07	5 0	6 3	6 1	4 0
,08	5 3	7 3	7 1	4 2
,09	6 2	8 3	8 0	5 0
,10	7 1	9 2	9 0	5 2

TABLE

FOR REDUCING THE CURRENCIES, &c. CONTINUED.

		New-Hamp. &c. &c.				New-York, &c.				New-Jersey, &c.				S. Carolina, &c.			
Doll.	*cts.*	£.	*s.*	*d.*	*q.*	£.	*s.*	*d.*	*q.*	£.	*s.*	*d.*	*q.*	£.	*s.*	*d.*	*q.*
	,20		1	2	2		1	7	1		1	6	0			11	1
	,30		1	9	2		2	4	3		2	3	0		1	4	3
	,40		2	4	3		3	2	2		3	0	0		1	10	2
	,50		3	0	0		4	0	0		3	9	0		2	4	0
	,60		3	7	1		4	9	2		4	6	0		2	9	2
	,70		4	2	2		5	7	1		5	3	0		3	3	1
	,80		4	9	2		6	4	3		6	0	0		3	8	3
	,90		5	4	3		7	2	2		6	9	0		4	2	2
1			6	0	0		8	0	0		7	6	0		4	8	0
2			12	0	0		16	0	0		15	0	0		9	4	0
3			18	0	0	1	4	0	0	1	2	6	0		14	0	0
4		1	4	0	0	1	12	0	0	1	10	0	0		18	8	0
5		1	10	0	0	2	0	0	0	1	17	6	0	1	3	4	0
6		1	16	0	0	2	8	0	0	2	5	0	0	1	8	0	0
7		2	2	0	0	2	16	0	0	2	12	6	0	1	12	8	0
8		2	8	0	0	3	4	0	0	3	0	0	0	1	17	4	0
9		2	14	0	0	3	12	0	0	3	7	6		2	2	0	0
10		3	0	0	0	4	0	0	0	3	15	0		2	6	8	0
20		6				8				7	10	0		4	13	4	
30		9				12				11	5	0		7	0	0	
40		12				16				15	0	0		9	6	8	
50		15				20				18	15	0		11	13	4	
60		18				24				22	10	0		14	0	0	
70		21				28				26	5	0		16	6	8	
80		24				32				30	0	0		18	13	4	
90		27				36				33	15	0		21	0	0	
100		30				40				37	10	0		23	6	8	
200		60				80				75	0	0		46	13	4	
300		90				120				112	10	0		70	0	0	
400		120				160				150	0	0		93	6	8	
500		150				200				187	10	0		116	13	4	
600		180				240				225	0	0		140	0	0	
700		210				280				262	10	0		163	6	8	
800		240				320				300	0	0		186	13	4	
900		270				560				337	10	0		210	0	0	
1000		300				400				375	0	0		233	6	8	
2000		600				800				750				466	13	4	
3000		900				1200				1125				700	0	0	
4000		1200				1600				1500				933	6	8	
5000		1500				2000				1875				1166	13	4	
6000		1800				2400				2250				1400	0	0	
7000		2100				2800				2625				1633	6	8	
8000		2400				3200				3000				1866	13	4	
9000		2700				3600				3375				2100	0	0	
10000		3000				4000				3750				2333	6	8	

§ 5. SIMPLE INTEREST.

INTEREST is the allowance given for the use of money, by the borrower to the lender. It is computed at so many dollars for each hundred lent for a year, (*per annum*) and a like proportion for a greater or less time. The highest rate is limited by our laws to 6 *per cent*,* that is 6 dollars for a hundred dollars, 6 cents for a hundred cents, £6 for a £100, &c. This is called *legal interest*, and is always understood when no other rate is mentioned.

There are three things to be noticed in Interest.

1. The PRINCIPAL; or money lent.
2. The RATE; or sum *per cent.* agreed on.
3. The AMOUNT; or principal and interest added together.

Interest is of two sorts, *Simple* and *Compound*.

1. Simple Interest is that which is allowed for the principal only.

2. Compound Interest is that which arises from the interest being added to the principal, and (continuing in the hands of the lender) becomes a part of the principal at the end of each stated time of payment.

GENERAL RULE.

1. *For one year*, multiply the principal by the rate, from the product cut off the two right hand figures of the dollars, which will be cents, those to the left hand will be dollars; or, which is the same thing, remove the *separatrix* from its natural place two figures towards the left hand, then all those figures to the left hand will be dollars, and those to the right hand will be cents, mills, and parts of a mill.

In the same way is calculated the interest on any sum of money in pounds, shillings, pence and farthings, with this difference only, that the two figures cut off to the right hand of pounds, must be reduced to the lowest denomination, each time cutting off as at first.

2. *For two or more years*, multiply the interest of one year by the number of years.

3. *For months*, take proportional or aliquot parts of the interest for one year, that is, for 6 months, $\frac{1}{2}$; for 4 months, $\frac{1}{3}$; for 3 months, $\frac{1}{4}$, &c.

For days, the proportional or aliquot parts of the interest for one month, allowing 30 days to a month.

EXAMPLES.

1. What is the interest of $86,446 for one year, at 6 *per cent?*

OPERATION.

Dolls.	*cts.*	*mills.*
86	44	6 *principal.*
		6 *rate.*
5| 18	67	6 *interest.*

In the product of the principal multiplied by the rate is found the answer. Thus cutting off the two right hand figures from the dollars leave five on the left hand which is dollars; the two figures cut off (18) are cents, the next figure (6) is mills; all the figures which may chance to be at the right hand of mills, are parts of a mill; hence we collect the *Ans.* $5 18*cts.* $6\frac{76}{100}$*m*

* In New-York the law allows 7 per cent.

2. What is the interest of \$365 14*cts.* 6*mills*, for three years 7 months and 6 days?

OPERATION.

		3 6 5, 1 4 6	*principal.*
		6	*rate.*
6 months $\frac{1}{2}$)	2	1|9 0, 8 7 6	*interest for* 1 *year.*
		3	
		6 5, 7 2 6 2 8	*interest for* 3 *years.*
1 month $\frac{1}{6}$)		1 0, 9 5 4 3 8	*interest for* 6 *months.*
6 days $\frac{1}{5}$)		1, 8 2 5 7 3	*interest for* 1 *month.*
		, 3 6 5 1 4	*interest for* 6 *days.*
		\$7 8, 8 7 1 5 3	interest for 3 years, 7 months and 6 days; that is \$78 87*cts.* $1\frac{53}{100}$*mills.*

Because 7 months are not an even part of a year, take two such numbers as are even parts, and which added together will make 7 (6 and 1) 6 months are $\frac{1}{2}$ of a year, therefore for 6 months, divide the interest of one year by 2; again 1 month is $\frac{1}{6}$ of 6 months, therefore *for* 1 *month*, divide the interest of 6 months by 6. For the days, because 6 days are $\frac{1}{5}$ of a month, or of 30 days, therefore *for* 6 *days*, divide the interest of 1 month by 5. Lastly add the interest of all the parts of the time together, the sum is the answer.

3. What is the interest of £71 7*s.* 6$\frac{1}{2}$*d.* for 1 year at 6 per cent?

OPERATION.

£.	*s.*	*d.*	*q.*
71	7	6	2
			6
£4|28	5	3	0
20			
*s.*5|65			
12			
*d.*7|83			
4			
*q.*3|32			

Ans. £4 5*s.* 7$\frac{3}{4}$*d.*

4. What is the interest of 16*s.* 8*d.* for 1 year? *Ans.* 1*s.*

When the rate is at 6 per cent, there is not perhaps a more concise and easy way of casting interest, on any sum of money in Dollars, Cents, and Mills, than by the following

METHOD.

Write down half the greatest *even number* of months for a multiplier; if there be an odd month it must be reckoned 30 days, for which and the given days, if any, seek how many times you can have six in the sum of them, place the figure for a decimal at the right hand of half the even number of months, already found, by which multiply the principal; observing in pointing off the product to remove the decimal point or separatrix *two figures* from its natural place towards the left hand, that is, point off *two* more places for decimals in the product, than there are decimal places in the multiplicand and multiplier counted together; then all the figures to the left hand of the point will be dollars, and those to the right hand, dimes, cents and mills, &c. which will be the interest required.

Should there be a remainder in taking one sixth of the days, reduce it to a vulgar fraction, for which take aliquot parts of the multiplicand. Thus

If the remainder be	$1=\frac{1}{6}$,	divide the multiplicand	by 6
If - - - - - -	$2=\frac{1}{3}$,	- - - - - - - - -	by 3
If - - - - - -	$3=\frac{1}{2}$,	- - - - - - - - -	by 2
If - - - - - -	$4=\frac{2}{3}$,	- - - - - - - - -	by 3 twice.
If - - - - - -	$5=\frac{1}{2}$, and $\frac{1}{3}$,	- - - - - -	by 2 and 3.

The quotients which in this way occur, must be added to the product of the principal multiplied by half the months, &c. the sum thus produced will be the interest required.

When there are days, but a less number than 6, *so that* 6 *cannot be contained in them*, put a cypher in place of the decimal at the right hand of the months, then proceed in all respects as above directed.

NOTE. In casting interest, each month is reckoned 30 days.

EXAMPLES.

1 What is the interest of $76,54 for 1 year, 7 months and 11 days?

OPERATION.

	7 6, 5 4
	9, 6
	9 5 9 2 4
	6 8 8 8 6
$\frac{1}{2}$	3 8 2 7
$\frac{1}{3}$	2 5 5 7
Ans.	7, 4 1 1 6 2
	Dolls. cents. mills.

The number of months being 19, the greatest even number is 18, half of which is 9, which I write down; then seeking how often 6 is contained in 41, (the sum of the days in the odd month and given days) I find it will be 6 times, which I set down at the right hand of half the even number of months for a decimal, by which together I multiply the principal. In taking one sixth of the days (41) there will be a remainder of $5=\frac{1}{2}$ and $\frac{1}{3}$ for which I take, first *one half* the multiplicand, that is, divide the multiplicand by 2, then by 3, and these quotients added, with the products of half the even number of months, &c. the sum of them will shew the interest required, observing to count off *two* more figures for decimals in the product than there are decimal figures in both the multiplier and multiplicand counted together.

For the conciseness and simplicity of the above method it is conceived that instructors will recommend it to their pupils in preference to any other.

2. What is the interest of $5,93 for 2 years and 8 months?
Ans. 94*cts.* 8*m.*

3. What is the interest of $67,62 for 3 years and 2 months?
Ans. $12 84*cts.* 7*m.*

4. What is the interest of 91 cents for 27 years?
Ans. $1 47*cts.* 4*m.*

When the interest on any sum is required for a great number of years it will be easier first to find the interest for 1 year, then multiply the interest so found by the number of years.

5. What is the interest of $2870,32 for 10 days? *Ans.* $4 78*cts* 3*m.*

When the rate is any other than 6 *per cent*, first find the interest at 6 per cent, then divide the interest so found by such parts as the interest at the rate required exceeds or falls short of the interest at 6 per cent, and the quotient added to or subtracted from the interest at 6 per cent, as the case may be, will give the interest at the rate required.

6. What is the interest of $137, 84 for 2 years and 6 months, at 5 per cent? *Ans.* $17,23.

7. What is the interest of $79,07 for 10 months at 8 per cent? *Ans.* $5,271.

8. What is the interest of $2,29 for 1 month 19 days at 3 per cent?
Ans. 9 *mills.*

9. What is the interest of $18 for 2 years 14 days, at 7 per cent?
Ans. $2 56*cts.* 9*m.*

10. What is the interest of $1600 for 1 year and 3 months?
Ans. $120.

11. What is the interest of $5,811 for 1 year and 11 months?
Ans. 66*cts.* 8*m.*

12. What is the interest of $17,68 for 11 months and 28 days?
Ans. $1,054.

13. What is the interest of $861,12 for 9 months 25 days, at 7 per cent? *Ans.* $49,394.

14. What is the interest of $105 ,61 for 1 year 7 months and 6 days?
Ans $10 13*cts.* 8*m.*

15. What is the interest of $86 for 9 months? *Ans.* $3,87.

16. What is the interest of $78,36 for 5 years 10 months and 3 days?
Ans. $27 46*cts.* 5*m.*

17. What is the interest of $812 30 cents for 2 years 8 months and 4 days? *Ans.* $130,509.

To this mode of computing interest, I would add from the "*Massachusetts Justice*" a

METHOD

Of computing the interest due upon Bonds, Notes, &c. when partial payments may at different times be made, as established by the Courts of Law in Massachusetts.

RULE.

Cast the interest up to the first payment, and if the payment exceed the interest, deduct the excess from the principal, and cast the interest upon the remainder to the time of the second payment. If the payment be less than the interest, place it by itself, and cast on the interest to the time of the next payment, and so on until the payments exceed the interest, then deduct the excess from the principal and proceed as before.

EXAMPLES.

Suppose A should have a bond against B for 1166 dollars 66 cents and 6 mills, dated May 1, 1796, upon which the following payments should be made, viz

	Dolls. Mills.	*Months.*	*Days.*
1. December 25, 1796 - - - - - -	166,666	7	24
2. July 10, 1797 - - - - - - - - -	16,666	6	15
3. September 1, 1798 - - - - - -	50,000	13	21
4. June 14, 1799 - - - - - - - - -	333,333	9	13
5. April 15, 1800 - - - - - - - - -	620,000	10	1
What will be due upon it August 3, 1801?		15	18

Ans. $237,76.

To facilitate the operation, let the space of time from the date of the Bond to the day of the first payment, and from the time of one payment to that of another, and from that of the last payment to the time of settlement, be first computed and set down against the day of payment as above.—

Then set down the sum on which the interest is to be cast, with the interest and payments in columns thus.

	Principal.	Time.		Interest.	Payments.	Excess.
	Dolls. Mills.	*Mo.*	*Da.*	*Dolls. M.*	*Dolls. M.*	*Dolls. M.*
1	1166,666	7	24	45,499	166,666	121,167
	121,167					
2	1045,499	6	15	33,978	16,666	
3	1045,499	13	21	71,616	50,000	
4	1045,499	9	13	49,312	333,333	
	245,093			154,906	399,999	245,093
5	300,406	10	1	40,153	620,000	579,847
	579,847					
	220,559	15	18	17,203		

The last remainder	220,559
Interest from the last payment	17,203
Sum due August 3, 1801	237,762

2. Supposing a note of 867 dollars 33 cents, dated January 6, 1794, upon which the following payments should be made, viz.

1. April 16, 1797	$136,44cts.
2. April 16, 1799	319,
3. Jan. 1, 1800	518,68

What would be due July 11, 1801? *Ans.* $215,103.

SUPPLEMENT TO SIMPLE INTEREST.

QUESTIONS.

1. What is Interest?
2. What is understood by 6 per cent? 3 per cent? 8 per cent, &c
3. What per cent per annum is allowed by law to the lender for the use of his money?
4. What is understood by the principal? the rate? the amount?
5. Of how many kinds is interest? in what does the difference consist
6. How is simple interest calculated for one year in Federal Money?
7. For more years than one, how is the interest found?
8. When there are months and days, what is the method of procedure?
9. What other method is there of casting interest on sums in Federal Money?
10. When the days are a less number than 6, so that 6 cannot be contained in them, what is to be done?
11. How is simple interest cast in pounds, shillings, pence and farthings:
12. When partial payments are made at different times, how is the interest calculated?

EXERCISES.

1. What is the interest of $916,72 for 1 year and 4 months? *Ans.* $73,337.

2. What is the interest of $93, 17*cts.* for 11 days? *Ans.* 17 *cents.*

3. What is the interest of $5,19 for 7 months? *Ans.* 18*cts.* 1*m.*

4. What is the interest of $1,07 for 3 years, 6 months and 15 days? *Ans.* 22*cts* 7*m.*

5. What is the interest of two hundred dollars and six cents, 4 days? *Ans.* 13*cts.* 3*m.*

6. What is the interest of nine cents 45 years 7 months and 11 days? *Ans.* 24*cts.* 6*m.*

7. What is the interest of half a mill 567 years? *Ans.* 1*ct.* 7*m.*

8 A's note of $365,37 was given Dec. 3, 1797; June 7, 1800 he paid $97,16; what was there due Sept. 11, 1800? *Ans.* $328,32.

9. B's note of $175 was given Dec. 6, 1798, on which was endorsed one year's interest: what was there due Jan. 1, 1803? *Ans.* $207,22.

10. C's note of $56,75 was given June 6, 1801, on interest after 90 days; what was there due Feb. 9, 1802? *Ans.* $58,19.

11. D's note of two hundred three dollars and seventeen cents was given Oct. 5, 1808, on interest after 3 months; Jan. 5, 1809, he paid fifty dollars; what was there due May 2d, 1811? *Ans.* $174, 53.

12. E's note of eight hundred seventy dollars and five cents, was given Nov. 17, 1800 on interest after 90 days; Feb. 11, 1805 he paid one hundred eighty six dollars and six cents; what was there due Dec. 23, 1807? *Ans.* $1045. 34.

13. What is the interest of £41 11s. $3\frac{1}{4}d.$ for a year and 2 months?

Ans. £2 18s. $2\frac{1}{4}d.$

14. What is the interest of $273,51, at 7 per cent for 1 year and 10 days?

Ans. $19,677.

15. Supposing a note of $317,92, dated July 5, 1797, on which were the following payments—Sept. 13, 1799, $208,04; March 10, 1800, $76, what was the sum due Jan. 1, 1801?

Ans. $83,991.

COMPOUND INTEREST

Is calculated by adding the Interest to the principal at the end of each year, and making the amount the principal for the succeeding year; then the given principal subtracted from the last amount, the remainder will be the compound interest.

A concise and easy method of casting Compound Interest, at 6 *per cent on any sum in Federal Money.*

RULE.

Multiply the given sum, if

For 2 years by 112,36	For 7 years by 150,3630
3 years — 119,1016	8 years — 159,3848
4 years — 126,2476	9 years — 168,9478
5 years — 133,8225	10 years — 179,0847
6 years — 141,8519	11 years — 189,8298

NOTE 1. Three of the first highest decimals in the above numbers will be sufficiently accurate for most operations; the product remembering to remove the separatrix *two* figures from its natural place towards the left hand, will then shew the amount of principal and compound interest for the given number of years. Subtract the principal from the amount and it will shew the compound interest.

2. *When there are months and days;* first find the amount of principal and compound interest for the years, agreeable to the foregoing method, then for the months and days cast the simple interest on the amount thus found; this added to the amount will give the answer.

3. Any sum of money at Compound Interest, will double itself in 11 years 10 months and 22 days.

EXAMPLES.

1. What is the compound interest of $56,75 for 11 years?

OPERATION.

```
         5 6, 7 5
       1 8 9, 8 2 9
       ------------
         5 1 0 7 5
       1 1 3 5 0
     4 5 4 0 0
   6 1 0 7 5
 4 5 4 0 0
 5 6 7 5
---------------------
1 0 7, 7 2 7 9 5 7 5 Amount.
  5 6, 7 5 principal subtracted.
---------------------
$ 5 0, 9 7 compound interest.
```

2. What is the amount of $236 at compound interest for 4 years, 7 months and six days?

OPERATION.

```
      1 2 6, 2 4 7 6
                2 3 6
      ---------------
        7 5 7 4 8 5 6
      3 7 8 7 4 2 8
    2 5 2 4 9 5 2
    -----------------
    $2 9 7, 9 4 4 3 3 6 Amount for 4 yrs.
           3, 6
    -----------------
    1 7 8 7 6 6 4
    8 9 3 8 3 2
    -----------------
   $1 0, 7 2 5 9 8 4 interest for 7 mo. 6 days.
    2 9 7, 9 4 4 amount for 4 years added.
   ------------------
   $3 0 8, 6 6 9
```

§ 6. COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION is when the Multiplicand consists of several denominations. It is particularly useful in finding the value of Goods.

The different denominations in what was formerly called *Lawful Money*, render this rule with some others in Arithmetic, as *Compound Division* and *Practice*, rules of great usefulness, quite tedious, and the variety of cases necessarily introduced, extremely burthensome to the memory.—This *lumber* of the mind might be almost wholly dispensed with, were the habit of reckoning in *Federal Money* generally adopted throughout the United States.

For important reasons, *pounds*, *shillings*, *pence* and *farthings*, ought to fall wholly into disuse: Federal Money is our national currency; the scholar might encompass the most useful rules in Arithmetic in half the time; the value of commodities bought and sold, might be cast with half the trouble, and with much less liability to errors, were all the calculations in money universally made in *Dollars*, *Cents*, and *Mills*. But this, to be practised, must be taught; it must be taught in our schools, and so long as the prices of goods, and almost every man's accounts are in *Pounds*, *Shillings*, *Pence* and *Farthings*, this mode of reckoning must not be left untaught.

To comprise the greater usefulness, and also to shew the great advantage which is gained by reckoning in Federal Money, I have contrasted the two modes of account, and in separate columns on the same page, have put the same questions in Old Lawful and in Federal Money.

OPERATIONS.

IN POUNDS, SHILLINGS, PENCE, FARTH.

CASE I.

When the quantity does not exceed 12 yards, pounds, &c. set down the price of one yard or pound, and place the quantity underneath the lowest denomination for a multiplier. Begin by multiplying the lowest denomination, and carry by the same rules from one denomination to another, as in Compound Addition.

EXAMPLES.

1. What will 7 yards of cloth cost at 9*s*. 5*d*. per yard?

OPERATION.

	£.	s.	d.	
	0	9	5	*price of* 1 *yard.*
			7	*yards.*
Ans.	3	5	11	*price of* 7 *yards.*

I say 7 times 5 is 35 pence=2*s*. 11*d*. I set down 11 and carry 2, saying 7 times 9 is 63, and 2 I carry are 65*s*. =£3 5*s*. which I set down.

IN DOLLARS, CENTS, MILLS.

IN ALL CASES.

Multiply the price and the quantity together, according to the rules of multiplication in Decimal Fractions, and the product will be the answer. That is,

Multiply as in Simple Multiplication, and from the product point off so many places for cents, and mills, as there are places of cents and mills in the price.

EXAMPLES.

1. What will 7 yards of cloth cost at $1,57 (equal to 9*s*. 5*d*.) per yard?

OPERATION.

	D.	cts.	
	1,	57	*price.*
		7	*quantity.*
Ans.	10	99	*price of* 7 *yards.*

As there are two decimal places in the price, so I make two in the product.

POUNDS, SHILLINGS, PENCE, FARTH.

2. What will 9 pounds of sugar cost at 10*d.* per lb.?

Ans. 7*s.* 6*d.*

3. What will 6 yards of cloth cost at £1 10*s.* 5*d.* per yard?

Ans. £9 2*s.* 6*d.*

CASE 2.

When the quantity exceeds 12, *and is any number within the Multiplication Table*, multiply by two such numbers as when multiplied together, will produce the given quantity.

If two numbers will not do this exactly, multiply by two such numbers as come the nearest to it, and by the deficiency or excess multiply the multiplicand, and this product added to or subtracted from the first product as the case may require, gives the answer.

EXAMPLES.

1. What will 42 yards of cloth cost at 15*s.* 9*d.* per yard?

OPERATION.

	£.	*s.*	*d.*	
	0	15	9	*price of* 1 *yd.*
Multiplied by			6	
	4	14	6	*price of* 6 *yds.*
Multiplied by			7	
Ans.	33	1	6	*price of* 42 *yds.*

Because 6 times 7 is 42, I multiply the price of 1 yard by 6, and this product by 7, as the rule directs.

DOLLARS, CENTS, MILLS.

2. What will 9 pounds of sugar cost at $0,139 per lb.? *Ans.* $1,251.

3. What will 6 yards of cloth cost at $5,07 per yard?

Ans. $30,42.

4. What will 42 yards of cloth cost at $2,625 per yard?

OPERATION.

	D.	*cts.*	*m.*
	2,	6 2	5
		4	2
	5	2 5	0
	1 0 5	0 0	0
Ans.	1 1 0	2 5	0

POUNDS, SHILLINGS, PENCE, FARTH.

2. What will 125 yards of cloth cost at 5*s.* 7*d.* per yard?

Ans. £34 17*s.* 11*d.*

3. What will 51 pounds of tea cost at 3*s.* 6*d.* per lb.?

Ans. £8 18*s.* 6*d*

4. What will 130 yards of cloth cost at £2 3*s.* 9*d.* per yard?

Ans. £284 7*s.* 6*d.*

CASE 3.

When the multiplier, that is, the quantity, exceeds 144, multiply first by 10, and this product again by 10, which will give the price of 100 yards, &c. and if the quantity be even hundreds, multiply the price of 100 by the number of hundreds in the question, and the product will be the answer; if there be odd numbers, multiply the price of 10 by the number of tens, and the price of unity, or 1, by the number of units, then these several products added together will be the answer.

DOLLARS, CENTS, MILLS.

5. What will 125 yards of cloth cost at 93 cents per yard?

Ans. $116,25.

6. What will 51 pounds of tea cost at $0,583 per lb.?

Ans. $29,733.

7. What will 130 yards of cloth cost at $7,25 per yard?

Ans. $942,50.

POUNDS, SHILLINGS, PENCE, FARTH.

EXAMPLES.

1. What will 563 yards of cloth cost at £1 6s. 7d. per yard?

OPERATION.

	£.	s.	d.	
	1	6	7	*price of* 1 *yard.*
			10	
	13	5	10	*price of* 10 *yds.*
			10	
	132	18	4	*price of* 100 *yds.*
			5	
	664	11	8	*price of* 500 *yds.*
6 times 10 yds.	79	15	0	*price of* 60 *yds.*
3 times 1 yd.	3	19	9	*price of* 3 *yds.*
Ans.	748	6	5	*price of* 563 *yds.*

2. What will 328 yards of cloth cost at 10s. 6½d. per yard?

Ans. £172 17s. 8d.

3. What will 624 yards of cloth cost at 12s. 8d. per yard?

Ans. £395 4s.

DOLLARS, CENTS, MILLS.

8. What will 563 yards of clo cost at $4,43 per yard?

OPERATION.

```
   Yds. 5 6 3
       $4, 4 3
   -----------
        1 6 8 9
      2 2 5 2
    2 2 5 2
   -------------
  $ 2 4 9 4, 0 9 Ans.
```

9. What will 328 yards of cl cost at $1,757 per yard?

Ans. $576,296.

10. What will 624 yards of cl cost at $2,111 per yard?

Ans. $1317,264

SUPPLEMENT TO COMPOUND MULTIPLICATION.

QUESTIONS.

1. What is Compound Multiplication?
2. What is its use?
3. Are operations more easy in OLD LAWFUL or FEDERAL MONEY?
4. What is the Rule of Compound Multiplication?
5. When the quantity, that is the Multiplier, exceeds 12, and is within the Multiplication Table, what are the steps to be taken?
6. When no two numbers multiplied together will produce the given quantity, what then is to be done?
7. When the multiplier exceeds 144, what is the method of procedure?
8. When the price of goods are given in Federal Money, what is the general and universal rule for finding their value by Multiplication?

EXERCISES.

1. A man has 38 silver cups, each weighing 1*oz.* 3*pwts.* 16*grs.* how much silver do they all contain?
Ans. 3 *lb.* 8 *oz.* 19 *pwts.* 8 *grs.*

2. If a man travel 34 miles, 3 furlongs, and 17 rods in one day, how far will he travel in 62 days?
Ans. 2134 *miles*, 4 *fur.* 14 *rods.*

3. What will 235 yards of cloth come to at £1 2*s.* 5½*d.* per yard?
Ans. £263 17*s.* 8½*d.*

4. If a horse run a mile in 12 minutes, 16 seconds, in what time would he go 176 mlies?
Ans. 1*D.* 11*h.* 58*m.* 56*sec*

§ 7. COMPOUND DIVISION.

COMPOUND DIVISION is the dividing of different denominations.

OPERATIONS.

IN POUNDS, SHILLINGS, PENCE, FARTH.

CASE 1.

1. *When the divisor, that is, the quantity, does not exceed* 12, begin at the highest denomination, and in the manner of short division, find how many times the divisor is contained in it; place the quotient under its own denomination, and if any thing remain, reduce it to the next less denomination, and divide as before, so proceed through all the denominations.

2. *If the quantity exceed* 12, *and there be any two numbers which multiplied together will produce it*, divide the price first by one of those numbers, and this quotient by the other.

EXAMPLES.

1. If 5 yards of cloth cost £3 13*s*. 6*d*. what is that per yard?

OPERATION.

£.	*s*.	*d*.	
5)3	13	6	*price of* 5 *yards*.
0	14	$8\frac{1}{4}$	*price of* 1 *yard*.

Finding I cannot have the divisor (5) in the first denomination (£3) I reduce it to shillings, (60) and add in the 13 shillings, which make 73 shillings in which the divisor (5) is contained 14 times and 3 remain; I set down the 14, and the remainder (3 shillings) reduce to pence (36) and the 6*d*. added make 42 pence in which the divisor is contained eight times and *two* remain; I set down the 8 and reduce the 2 pence to farthings (8) in which I have the divisor *once* (1 qr. or $\frac{1}{4}d.$) and a remainder of $\frac{3}{5}$ of a farthing, which being of small value is neglected.

2. If 48 yards of cloth cost £4 16*s*. $4\frac{1}{2}d.$ what is that per yard?

Ans. £0 2*s*.

IN DOLLARS, CENTS, MILLS.

IN ALL CASES.

Divide the price by the quantity, and point off so many places for cents and mills in the product as there are places of cents, and mills in the dividend.

If the quantity be a composite number, that is produced by the multiplication of two numbers, the operation may be varied by dividing the price first by one of those numbers, and this quotient by the other.

EXAMPLES.

1. If 5 yards of cloth cost $12,25, what is that per yard?

OPERATION.

D.	*Cts*.
5)12,	25
Ans. 2,	45

There are two decimal places in the dividend. I therefore point off two places for decimals or cents in the quotient.

2. If 48 yards of cloth cost $16,06, what is that per yard?

Ans. $0 33 cents.

POUNDS, SHILLINGS, PENCE, FARTH.

3. If 24*lb.* of tea cost £2 7*s.* 9¾*d.* what is that per *lb.*?

Ans. £0 1*s.* 11¾*d.*

4. If 35 yards of cloth cost £42 6*s* 7½*d.* what is that per yard?

Ans. £1 4*s.* 2¼*d.*

CASE 2.

1. "*Having the price of a hundred weight* (112*lb.*) *to find the price of* 1*lb.* divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1*lb.* required."

2. *If the number of hundred weight be more than one*, first divide the whole price by the number of hundreds, then proceed as before.

EXAMPLES.

1. If 1*cwt.* of sugar cost £3 7*s.* 6*d.* what is that per *lb.*?

OPERATION.

	£.	s.	d.	q.	
8)	3	7	6		*price of* 1*cwt.*
7)	0	8	5	1	*price of* 14*lb. or* ⅛*cwt.*
2)		1	2	2	*price of* 2*lb. or* $\frac{1}{56}$*cwt.*
Ans.	0	7	1		*price of* 1*lb.*

DOLLARS, CENTS, MILLS.

3. If 24*lb.* of tea cost $7,97 what is that per *lb.*?

Ans. $0,233.

4. If 35 yards of cloth cost $141,103, what is that per yard?

Ans. $4,031.

The same may be done in Federal Money.

5. If 1 *cwt.* of sugar cost $11,25, what is that per *lb.*?

Ans. 10 *cents.*

POUNDS, SHILLINGS, PENCE, FARTH.

2. If 8*cwt.* of cocoa cost £15 7*s.* 4*d.* what is that per lb.? *Ans.* 4*d.*

3. If 3*cwt.* of sugar cost £15 13*s.* what is that per lb.?

Ans. 11*d.*

CASE 3.

"*When the divisor is such a number as cannot be produced by the multiplication of small numbers,* divide after the manner of long division, setting down the work of dividing and reducing."

DOLLARS, CENTS, MILLS.

6. If 8*cwt.* of cocoa cost $51,223, what is that per lb.?

Ans. 5*cts.* 7*m.*

7. If 3*cwt.* of sugar cost $52,16 what is that per lb.?

Ans. 15*cts.* 5*m.*

POUNDS, SHILLINGS, PENCE, FARTH.

EXAMPLES.

1. If 46 yards of cloth cost £53, s. 6d. what is that per yard?

OPERATION.

```
    £.   s.  d.  £.   s.  d.
46)53   10  6( 1    3   3¼ Ans.
   46
   —
    7
   20
   ——
46)150(3
   138
   ———
    12
    12
   ———
46)150(3
   138
   ———
    12
     4
   ———
 46)48(1
    46
    —
     2
```

2. If 263 bushels of wheat cost 6 7s. 10d. what is that per bushel? *Ans.* 6s. 6¾d.

3. If 670 gallons of wine cost 47 1s. 11d. what is that per gallon? *Ans.* 4s. 4½d.

DOLLARS, CENTS, MILLS.

8. If 46 yards of cloth cost $178,416, what is that per yard? *Ans.* $3,878.

9. If 263 bushels of wheat cost $287,973, what is that per bushel? *Ans.* $1,094.

10. If 670 gallons of wine cost $490,32, what is that per gallon? *Ans.* $0,73.

SUPPLEMENT TO COMPOUND DIVISION.

QUESTIONS.

1. What is Compound Division?
2. When the price of any quantity not exceeding 12, of yards, pounds, &c. is given in pounds, shillings, pence and farthings, how is the price of one yard found?
3. When the quantity is such a number as cannot be produced by the multiplication of small numbers, what is the method of procedure?
4. Having the price of an hundred weight given, in what way is found the price of 1 lb.?
5. If there be several hundred weight, what are the steps of operating?
6. When the price is given in Federal Money, what is the method of operating?

EXERCISES.

POUNDS, SHILLINGS, PENCE, FARTH.

1. If 10 sheep cost £4 5*s.* 7*d.* what is the price of each?

Ans. 8*s.* 6½*d.*

2. If 84 cows cost £253 13*s.* what is the price of each?

Ans. £3 0*s.* 4¼*d.*

DOLLARS, CENTS, MILLS.

Let the Scholar reduce the price of sheep and of the cows to Federal Money, and perform the operations in Dollars, Cents and Mills.

Price of 1 sheep $1,426.

Price of 1 cow, $10,065.

3. If 121 pieces of cloth measure 2896 yards, 1 qr. 3 na. what does each piece measure?

Ans. 23*yds.* 3*qr.* 3*na.*

4. If 66 tea-spoons weigh 9 10oz. 14pwt. what is the weight each?

Ans. 10*pwt.* 12$\frac{4}{11}$*grs.*

5. If 2cwt. of rice cost £2 11*s.* 8$\frac{1}{2}$*d.* what is that per lb.?

Ans. 2$\frac{1}{4}$*d.*

6. At £2 11*s.* 6$\frac{1}{2}$*d.* for 2cwt. rice, what is that in Federal Mone and what is that per lb.?

Price of 1*lb.* 3*cts.* 8*m.*

7. If 47 bags of indigo weigh 12cwt. 1qr. 26lb. 4oz. what does each weigh?

Ans. 1*qr.* 1*lb.* 12*oz.*

8. If 8 horses eat 900 bushels a peck of oats in 1 year, how mu will each horse eat per day?

Ans. 1*pk.* 1*qt.* 1*pt.* 2*gills.*

Divide £297 2s. 3d. among 4 men, 6 boys, and give each man 3 times so much as one boy; what will each man share, and each boy?

OPERATION.

The men have triple shares, therefore multiply the number of men (4) by 3, and add the number of boys, (6) for a divisor.

men.	*boys.*
4 &	6
3	
12	
6	
18 *the number of equal shares in the whole.*=Divisor.	

£.	s.	d.
18)297	2	3
18		
117		
108		
9		
20		
)182(10		
18		
2		
12		
)27(1		
18		
9		
4		
)36(2		
36		

	£.	s.	d.	q.	
	(16	10	1	2	=1 *boy's share.*
				3	
Ans.	49	10	4	2	=1 *man's share.*

PROOF.

£.	s.	d.	q.	
£49	10	4	2	
			4	
198	1	6	0	*men's share.*
16	10	1	2	and
			6	
99	0	9	0	*boys' share.*
£297	2	3	0	*added.*

10. Divide £39 12s. 5d. among 4 men, 6 women, and 9 boys; give each man double to a woman, each woman double to a boy.

	£.	s.	d.	
Ans.	1	1	5	*a boy's share.*
	2	2	10	*a woman's share.*
	4	5	8	*a man's share.*

§ 8. SINGLE RULE OF THREE.

THE Single Rule of Three, sometimes called the RULE OF PROPORTION is known by having three *terms* given to find the *fourth.*

It is of *two* kinds, *Direct* and *Indirect*, or *Inverse.*

SINGLE RULE OF THREE DIRECT.

The Single Rule of Three Direct teaches, by having three numbers given to find a fourth, which shall bear the same proportion to the third that the second does to the first.

It is evident, that the value, weight and measure of any commodity is proportionate to its quantity, that the amount of work, or consumption is proportionate to the time; that gain, loss and interest, when the time is fixed, is proportionate to the capital sum from which it arises; and that the effect produced by any cause is proportioned to the extent of that cause.

These are cases in direct proportion, and all others may be known to be so, when the number sought increases or diminishes along with the term from which it is derived. Therefore,

If *more* require *more*, or *less* require *less*, the question is always known to belong to the Rule of Three Direct.

More requiring more, is when the third term is greater than the first, and requires the fourth term to be greater than the second.

Less requiring less, is when the third term is less than the first and requires the fourth term to be less than the second.

RULE.

"1. State the question by making that number which asks the question, "the third term, or putting it in the third place; that which is of the "same name or quality as the demand, the first term, and that which is of "the same name or quality with the answer required, the second term."

"2. Multiply the second and third terms together, divide by the first, "and the quotient will be the answer to the question, which (as also the "remainder) will be in the same denomination in which you left the second "term, and may be brought into any other denomination required."

The chief difficulty that occurs in the *Rule of Three*, is the right placing of the numbers, or stating of the question; this being accomplished, there is nothing to do, but to multiply and divide, and the work is done.

To this end the nature of every question must be considered, and the circumstances on which the proportion depends, observed, and common sense will direct this if the terms of the question be understood.

The method of proof is by inverting the order of the question.

Note 1. If the first and third terms, both or either, be of different denominations, both terms must be reduced to the lowest denomination mentioned in either, before stating the question.

2. If the second term consists of different denominations, it must be reduced to the lowest denomination; the fourth term or answer will then be found in the same denomination, and must be reduced back again to the highest denomination possible.

3. After division if there be any remainder, and the quotient be not in the lowest denomination, it must be reduced to the next less denomination, dividing as before. So continue to do till it is brought to the lowest denomination, or till nothing remains.

4. In every question there is a supposition and a demand; the supposition is implied in the two first terms of the statement, the demand in the third.

5. When any of the terms are given in *Federal Money* the operation is conducted in all respects as in simple numbers, observing only to place the point or separatrix between dollars and cents, to point off the results according to what has been taught already in *Decimal Fractions, Federal Money*, and further illustrated in *Compound Division*.

6. When any number of barrels, bales, or other packages, or pieces are given, if they be of equal contents, find the contents of one barrel or piece, &c. in the lowest denomination mentioned, which multiply by the number of pieces, &c. the product will be the contents of the whole—If the pieces &c. be of unequal contents, find the content of each, add these together, and the sum of them will be the whole quantity.

7. The term which asks the question, or that which implies the demand, is generally known by some of these words going before it; How much? How many? How long? What cost? What will? &c.

EXAMPLES.

1. If 9*lbs.* of tobacco cost 6*s.* what will 25 *lbs.* cost?

OPERATION.

```
 lbs.  s.  lbs.
As 9 : 6 : : 25 to the answer.
      25
      —
      30
      12
     ——— s. d.
  9)150(16 8 answer.
    9
   ———
    60
    54
   ———
     6
    12
   ———
  9)72(8
    72
   ———
    00
```

Here 25*lbs.* which asks the question, (*what will* 25*lbs.* *&c.*) is made the third term, by being put in the third place; 9*lbs.* being of the same name, the first term, and 6*s.* of the same name with the term sought, the second term.

I multiply the second and third terms together, and divide by the first. The remainder (6) I reduce to pence, and divide as before. The quotients make the answer 16*s.* 8*d.*

By inverting the order of the question it will stand thus,

2. If 6*s.* buy 9*lbs.* of tobacco, what will 16*s.* 8*d.* buy?

```
 s.          s.  d.
 6           16  8
12           12
—            —
72 pence.   200 pence.

 pence.  lbs.  pence.
As 72     9 : : 200
              200
             ———
       72)1800(25lb. answer.
           144
           ———
            360
            360
            ———
```

Here the term which asks the question (16*s.* 8*d.*) is of different denominations; it must, therefore, be reduced to the lowest denomination mentioned (*pence*) as must also the other term of the same name, consequently, to be the first term.

Again—By inverting the order of the question.

3. If 16*s.* 8*d.* (=200 *pence*) buy 25*lbs.* of tobacco, how much will 6*s.* (=72 *pence*) buy?

OPERATION.

```
      d.      lbs.     d.
As 200  :  25  : :  72
             72
            ----
             50
            175
           -----
2|00)18|00(9lbs. Ans.
     18
    ----
```

These three questions are only the first varied; they shew how any question in this rule may be inverted.

4. If 1*oz.* of silver cost 6*s.* 9*d.* what will be the price of a silver cup that weighs 9*oz.* 4*pwt.* 16*grs.*?

NOTE.—As each of the terms contain different denominations, they must all be reduced to the lowest denomination mentioned.

Ans. 747 *pence*, $3\frac{3}{5}q$. which must be reduced to the highest denomination, thus,

```
   pence.
12)747 Rem. 3d.
   ----
20)62 Rem. 2s.
   ----
   £3 2s. 3d. 3⅗q. Ans.
```

5. If 6 horses eat 21 bushels of oats in 3 weeks, how many bushels will 20 horses eat in the same time? *Ans.* 70 *bushels.*

The same question inverted.

6. If 20 horses eat 70 bushels of oats in 3 weeks, how many bushels will 6 horses eat in the same time? *Ans.* 21 *bushels.*

The statement of every question requires thought and consideration;—here are *four* numbers given in the question; to know which three are to be employed in the statement, there can be no difficulty if the scholar proceed deliberately and as his rule directs—first consider which of the given numbers it is that asks the question; that determined on, put it in the third place, then seek for another number of the same name, or kind, put that in the first place, the second place must now be occupied by that number which is of the same name or kind with the number sought; when these steps are cautiously followed, the scholar cannot fail to make his statement right.

7. If an ingot of silver weigh 36*oz.* 10*pwt.* what is it worth at 5*s.* per ounce? *Ans.* £9 2*s.* 6*d.*

8. A Goldsmith sold a Tankard for £10 12*s.* at the rate of 5*s.* 4*d.* per ounce, I demand the weight of it. *Ans.* 39*oz.* 15*pwt.*

9. If the moon move 13*deg.* 10*min.* 35*sec.* in one day; in what time does it perform one revolution? *Ans.* 27*days.* 7*hrs.* 43*min.*

10. If a family of 10 persons spend 3 bushels of malt in a month, how many bushels will serve them when there are 30 in the family?
Ans. 9 *bushels*

11. If a family of 30 persons spend 9 bushels of malt in a month, how many bushels will serve a family of 10 persons the same time?
Ans. 3 *bushels.*

12. If 12 acres 3 roods, produce 78 quarters 3 pecks, how much will 35 acres, 1 rood, 20 poles produce?
Ans. 216 *qrs.* 5 *bush.* $1\frac{1}{2}$ *pecks.*

13. If 5 acres, 1 rood produce 26 quarters, 2 bushels, how many acres will be required to produce 47 quarters, 4 bushels? *Ans.* 9 *acres*, 2 *roods*

14. If 365 men consume 75 barrels of provisions in 9 months, how much will 500 men consume in the same time? *Ans.* 102$\frac{54}{73}$ *barrels.*

15. If 500 men consume 102$\frac{54}{73}$ barrels of provisions in 9 months, how much will 365 men consume in the same time?

OPERATION.

```
             barrels.
              102 54/73
Multiplied by  73   the denominator
             -----  of the fraction.
              306
             714
         Add  54  the numerator.
             ----
As 500 : 7500 :: 365
                 7500
                ------
                182500
                2555
               --------
           5|00)27375|00
              73)5475(75 Ans.
                 511
                 ---
                 365
                 365
```

NOTE. In the 15th example, in order to embrace the fraction ($\frac{54}{73}$ *of a barrel*) the integers 102 barrels must be multiplied by the denominator of the fraction (73) and the numerator, (54) added to the product.

After division, the quotient must be divided by the denominator of the fraction, and this last quotient will be the answer, all which may be seen in the example.

The Scholar must remember to do the same in all similar cases.

16. How much will 4 pieces of linen containing, viz. 35½, 36, 37½, and 38 yards come to at 79 cents per yard? *Ans.* $116,13.

7. If I give $6 for the use of 00 for 12 months, what must I e for 357,82 the same time?

Ans. $21,469.

18. How many tiles of 8 inches *square will lay a floor 20 feet long, and 16 feet broad?

Ans 720.

*Square your feet & your 6

9. If 2*lb.* of sugar cost 25 *cents*, at will 100*lb.* of coffee cost, if 8*lb.* ugar are worth 5*lb.* of coffee?

Ans. $20.

20. If £3 sterling be equal to £4 N. England currency, how much N. England currency will be equal to £1000 sterling?

Ans. £1333 6*s.* 8*d.*

1. If I buy 7*lb.* of sugar for 75 s, how much can I buy for 6 rs?

Ans. 56*lb.*

N. B. Sums in Federal Money are of the same denomination when the decimal places in each are equal.

To reduce sums in Federal Money to the same denomination, annex so many cyphers to that sum which has the least number of decimal places, or places of cents, mills, &c. as shall make up the deficiency.

22. If I buy 76 yards of cloth for $113,17, what did it cost per Ell English? *Ans.* $1,861.

23. A man spends $3,25 per week, what is that per annum? *Ans.* $169,464.

24. If 3 horses and 4 oxen be worth 9 cows, how many cows will 6 horses and 8 oxen be worth? *Ans.* 18.

25. Bought a silver cup, weighing 9*oz.* 4*pwt.* 16*grs.* for £3 2*s.* 3*d.* $3\frac{3}{8}q$ what was that per ounce? *Ans.* 6*s.* 9*d.*

26. There is a cistern which has 4 cocks, the first will empty in 10 minutes, the second in 20 minutes, the third in 40 minutes, and the fourth in 80 minutes; in what time will all four running together empty it?

$$\begin{array}{l} \textit{Min.} \\ \left\{\begin{array}{l}10\\20\\40\\80\end{array}\right. \end{array} \begin{array}{l} \textit{Cist.} \\ : 1 \end{array} \begin{array}{l} \\ :: \end{array} \begin{array}{l} \textit{Min.} \\ 60 : \end{array} \begin{array}{r} \textit{Cist.} \\ \left\{\begin{array}{r}6\\3\\1,5\\,75\end{array}\right. \end{array}$$

In 1 hour the 4 cocks would empty - - - - 11,25 *Cist.*

Then,

Cist.		*Min.*		*Cist.*		*Min.*
As 11,25	:	60	: :	1	·	5,33 *Ans.*

27. A man having a piece of land to plant, hired two men and a boy to plant it, one of the men could plant it in 12 days, the other in 15 days, and the boy in 27 days; in how long time would they plant it if they all worked together?

Ans. 5,346 *days.*

28. A merchant bought 270 quintals of cod fish, for $780; freight $37,70; duties and other charges $30,60; what must he sell it at per quintal to gain $143 in the whole?

Ans. $3,671.

The sum of all the expenses of the fish with the Merchant's gain must be found for the second term.

29. If a staff 5*ft.* 8*in.* in length cast a shadow of 6 feet; how high is that steeple whose shadow measures 153 feet? *Ans.* 144½ *feet.*

30. Bought 12 pieces of cloth each 10 yards, at $1,75 per yard, what came they to?

Ans. $210.

31. Bought 4 pieces of Holland, each containing 24 Ells English, for $96; how much was that per yard?

Ans. 80 *cents*

32. Bought 9 chests of tea, each weighing 3*C.* 2*qrs.* 21*lb.* at £4 9*s.* *per cwt.* what came they to?

Ans. £147 13*s.* $8\frac{1}{4}$*d.*

33. A bankrupt owes in all 972 dollars, and his money and effects are but \$607,50; what will a creditor receive on \$11,333? *Ans.* \$7,083

34. Bought 126 gallons of rum for \$110, how much water must be added to it to reduce the first cost to 75 cents per gallon? *Ans.* 20⅔ *gal.*

35. A owes B £3475, but B compounds with him for 13*s.* 4*d.* on the pound; what must he receive for his debt *Ans.* £2316 13*s.* 4*d.*

36. If a person whose rent is \$145 pays \$12,63 of parish taxes, how much should a person pay whose rent is \$378? *Ans.* \$32,925.

Inverse Proportion.

IN some questions the number sought becomes less, when the circumstances from which it is derived become greater. Thus when the price of goods increase the quantity which may be bought for a given sum, is smaller When the number of men employed at work is increased, the time in which they may complete it becomes shorter; and when the activity of any cause is increased, the quantity necessary to produce any given effect is diminished.

These and the like cases belong to the

SINGLE RULE OF THREE INVERSE.

The Single Rule of Three Inverse teaches by having three numbers given to find a fourth, having the same proportion to the second, as the first has to the third.

If more require less, or less require more, the question belongs to the Single Rule of Three Inverse.

More requiring less, is when the third term is greater than the first, and requires the fourth term to be less than the second.

Less requiring more, is when the third term is less than the first, and requires the fourth term to be greater than the second.

RULE.

"State and reduce the terms as in the rule of three direct; then multiply the first and second terms together, divide the product by the third, and the quotient will be the answer in the same denomination with the second term."

EXAMPLES.

1. If 48 men build a wall in 24 days, how many men can do the same in 192 days?

OPERATION.

```
 Men.  Days.  Men.
As 48 : 24 : : 192
        48
       ----
        192
        96
      ------
  192)1152(6 answer.
      1152
```

Here the third term is greater than the first, and common sense teaches the fourth term, or answer must be *less* than the second; for if 48 men can do the work in 24 days, certainly 192 men will do it in less time. In this way it may be determined if a question belong to the Rule of Three Inverse.

2. If a board be 9 inches broad, how much in length will make a square foot? *Ans.* 16 *inches.*

3. How many yards of sarcenet, 3*qrs.* wide, will line 9 yards of cloth of 8*qrs.* wide? *Ans.* 24 *yards.*

4. Lent a friend 292 dollars for 6 months; some time afterwards he lent me 806 dollars: how long may I keep it to balance the favor?

Ans. *2 months, 5 days.*

5. A garrison had provision for 8 months, at the rate of 15 ounces to each person per day; how much must be allowed per day in order that the provision may last $9\frac{1}{2}$ months?

Ans. $12\frac{12}{19}$ *ounces.*

6 A garrison of 1200 has provisions for 9 months at the rate of 14 ounces per day, how long will the provisions last at the same allowance if the garrison be reinforced by 400 men?

Ans. $6\frac{3}{4}$ *months.*

7.———How must the daily allowance be in order that the provisions may last 9 months after the garrison is reinforced?

Ans. $10\frac{1}{2}$ *ounces.*

8. How much land at $2,50 per acre should be given in exchange for 360 acres at $3,75 per acre?

Ans. 540 *acres.*

9. What sum should be put to interest to gain as much in 1 month as $127 would gain in 12 months?

Ans. $1524.

10. If a man perform a journey in 15 days, when the day is 12 hours long, in how many will he do it when the day is but 10 hours?
Ans. 18 *days.*

11. If a piece of land 40 rods in length, and 4 in breadth make an acre, how wide must it be when it is but 25 rods long?
Ans. $6\frac{2}{5}$ *rods.*

12. There was a certain building raised in 8 months by 120 workmen, but the same being demolished, it is required to be built in two months; I demand how many men must be employed about it? *Ans.* 480 *men*

13. How much in length, that is 3 inches broad, will make a square foot? *Ans.* 48 *inches.*

14. There is a cistern having 1 pipe which will empty it in 10 hours, how many pipes of the same capacity will empty it in 24 minutes?
Ans. 25 *pipes.*

15. If a field will feed 6 cows 91 days, how long will it feed 21 cows? *Ans.* 26 *days.*

16. If the quartern loaf weigh $4\frac{1}{2}$ pounds when wheat is $2 per bushel, what must it weigh when wheat is $1,50 the bushel?
Ans. 6*lb.*

17. How many yards of baize, 3 quarters wide, will line a cloak which has in it 12 yards of camblet, half yard wide?
Ans. 8 *yards.*

GENERAL RULE

For stating all questions whether direct or inverse.

1. Place that number for the third term, which signifies the same kind of thing, with what is sought, and consider whether the number sought will be greater or less. If greater, place the least of the other terms for the first; but if less, place the greater for the first, and the remaining one for the second term.

Multiply the second and third terms together, divide the product by the first, and the quotient will be the answer.

EXAMPLES.

1. If 30 horses plough 12 acres, how many will 40 plough in the same time?

OPERATIONS.

H. H. Ac.
30 : 40 : : 12
12
———
30)480(16 *Ans.*

Here because the thing sought is a number of acres, we place 12, the given number of acres, for the third term; and because 40 horses will plough more than 12, we make the lesser number, 30, the first term, and the greater number 40, the second term.

2. If 40 horses be maintained for a certain sum on hay at 5 cents per stone, how many will be maintained, on the same sum, when the price of hay rises to 8 cents per stone?

C. C. H.
8 : 5 : : 40
40
———
8)200(25 *Ans.*
16
—
40
40

Here, because a number of horses is sought, we make the given number of horses, 40, the third term, and because fewer will be maintained for the same money, when the price of hay is dearer, we make the greater price 8 cents, the first term, and the lesser price, 5 cents, the second.

The first of these examples is *Direct*, the second *Inverse.*

Every question consists of a supposition and a demand.

In the first the supposition is, that 30 *horses plough* 12 *acres*, and the demand *how many* 40 *will plough?* and the first term of the proposition, 30, is found in the supposition in this and every other *direct question.*

In the second, the supposition is that 40 *horses are maintained on hay at* 5 *cents per stone*, and the demand, *how many will be maintained on hay at* 8 *cents?* and the first term of the proportion, 8, is found in the demand, in this and every other inverse question.

3. If a quarter of wheat afford 60 tenpenny loaves, how many eightpenny loaves may be obtained from it? *Ans.* 75 *loaves.*

4. If in 12 months, 100 dollars gain 6 dollars interest, what will gain the same sum in 5 months? *Ans.* 240 *dollars*

SUPPLEMENT TO THE SINGLE RULE OF THREE.

QUESTIONS.

1. What is the Single Rule of Three; or the Rule of Proportion?
2. How many kinds of Proportion are there?
3. What is it that the Single Rule of Three Direct teaches?
4. How can it be known that a question belongs to the Single Rule of Three Direct?
5. What is understood by *more requiring more* and *less requiring less?*
6. How are questions in the Rule of Three stated?
7. Having stated the question, how is the answer found in direct proportion?
8. What do you observe of the first and third terms concerning the different denominations, sometimes contained in them?
9. When the second term contains different denominations, what is to be done?
10. How is it known what denomination the quotient is of?
11. If the quotient or answer be found in an inferior denomination, what is to be done?
12. When the terms are given in Federal Money, how is the operation conducted?
13. How are the sums in Federal Money reduced to the same denomination?
14. When any number of barrels, bales, pieces, &c. are given, what is the method of procedure?
15. What is it that the Single Rule of Three Inverse teaches?
16. How are the questions stated in Inverse Proportion?
17. What is understood by more requiring less and less requiring more?
18. How is the answer found in the Rule of Three Inverse?
19. What is the general Rule for stating all questions, whether Direct or Inverse?

EXERCISES.

. If my horse and saddle are worth 18 guineas, and my horse be worth six times so much as the saddle, pray what is the value of my horse?

Ans 72 dollars.

2. How many yards of matting that is half a yard wide will cover a room that is 18 feet wide and 30 feet long? *Ans.* 120 *yards.*

3. Suppose 800 soldiers were placed in a garrison, and their provisions were computed sufficient for two months; how many soldiers must depart that the provision may serve them 5 months? *Ans.* 480.

4. I borrowed 185 quarters of corn when the price was 19*s.* how much must I pay to indemnify the lender when the price is 17*s.* 4*d.*?

Ans. $202\frac{7}{52}$.

5. Bought 45 barrels of beef at $3,50 per barrel, among which are 16 barrels, whereof 4 are worth no more than 3 of the others; how much must I pay?

Ans. $143,50.

6. A and B depart for the same place and travel the same road; but A goes 5 days before B at the rate of 20 miles per day; B follows at the rate of 25 miles per day; in what time and distance will he overtake A?

Ans. B will overtake A in 20 days, and travel 500 miles.

Here two statements will be necessary, one to ascertain the time, and the other to ascertain the distance.

Method of assessing town or parish taxes.

1. An inventory of the value of all the estates, both real and personal, and the number of polls for which each person is rateable, must be taken in separate columns. Then to know what must be paid on the dollar, make the total value of the inventory the first term; the tax to be assessed the second; and 1 dollar the third, and the quotient will shew the value on the dollar.

NOTE. This method is taken from Mr. PIKE's Arithmetic, with this difference, that here the money is reduced to Federal Currency.

2. Make a table, by multiplying the value on the dollar by 1, 2, 3, 4, 5, &c.

3. From the Inventory take the real and personal estates of each man, and find them separately, in the table, which will shew you each man's proportional share of the tax for real and personal estates.

If any part of the tax be averaged on the polls, before stating to find the value on the dollar, deduct the sum of the average tax from the whole sum to be assessed; for which average make a separate column as well as for the real and personal estates.

EXAMPLES.

Suppose the General Court should grant a tax of 150000 dollars, of which a certain town is to pay $3250,72 and of which the polls being 624 are to pay 75 cents each; the town's inventory is 69568 dollars; what will it be on the dollar; and what is A's tax (as by the inventory) whose estate is as follows, viz. real, 856 dollars; personal, 103 dollars; and he has 4 polls?

Pol. Cts. Pol. Dolls.

1. As, 1 : ,75 : : 624 : 468 the average part of the tax to be deducted from $3250,72 and there will remain $2782,72.

Dolls. Dolls. Cts. Dolls. Cts.

2. As 69568 : 2782,72 : : 1 : 4 on the dollar.

TABLE.

Dolls.	*Dolls. cts.*	*Dolls.*	*Dolls. cts.*	*Dolls.*	*Dolls.*
1 is	4	20 is	80	200 is	8
2 —	8	30 —	1 20	300 —	12
3 —	12	40 —	1 60	400 —	16
4 —	16	50 —	2 00	500 —	20
5 —	20	60 —	2 40	600 —	24
6 —	24	70 —	2 80	700 —	28
7 —	28	80 —	3 20	800 —	32
8 —	32	90 —	3 60	900 —	36
9 —	36	100 —	4 00	1000 —	40
10 —	40				

Now to find what A's rate will be.

His real estate being 856 dollars, I find by the Table that 800 dollars is $32 cts.
that 50 — — 2
that 6 — — 0 24

Therefore the tax for his real estate is 34 24
In the like manner I find the tax for his personal estate to be 4 12
His 4 polls, at 75 cents each, are 3

$41 36

Real. *Dolls. Cts.*	Personal. *Dolls. Cts.*	Polls. *Dolls. Cts.*	Total. *Dolls. Cts.*
34 24	4 12	3 0	41 36

§ 9. DOUBLE RULE OF THREE.

THE Double Rule of Three, sometimes called COMPOUND PROPORTION, teaches by having five numbers given to find a sixth, which, if the proportion be *direct*, must bear the same proportion to the fourth and fifth as the third does to the first and second. But if the proportion be *inverse*, the sixth number must bear the same proportion to the fourth and fifth, as the first does to the second and third.

RULE.

1. "State the question, by placing the three conditional terms in such order that that number which is the cause of gain, loss, or action, may possess the first place; that which denotes space of time, or distance of place, the second; and that which is the gain, loss, or action, the third."

2. "Place the other two terms, which move the question, under those of the same name."

3. "Then, if the blank place, or term sought, fall under the third place, the proportion is direct, therefore, multiply the three last terms together, for a dividend, and the other two for a divisor; then the quotient will be the answer."

4. "But if the blank fall under the first or second place, the proportion is inverse, wherefore multiply the first, second and last terms together, for a dividend, and the other two, for a divisor; the quotient will be the answer."

EXAMPLES.

If 100 dollars gain 6 dollars in 12 months, what will 400 dollars gain in 8 months?

Statement of the question.

D.		M.		D.	
100	:	12	: :	6	*Terms in the supposition, or conditional terms.*
400	:	8			*Terms which move the question.*

Of the three conditional terms, it is evident that 100 dollars put at interest, is that one which is the cause of gain; consequently 100 dollars must be the first term; and because 12 months is the space of time in which the gain is made, this must be the second term; and 6 dollars which is the gain, the third term. The other two terms must then be arranged under those of the same name.

Now as the blank falls under the third place, therefore, the question is in direct proportion, and the answer is found by multiplying the three last terms together for a dividend, and the two first for a divisor.

OPERATION.

```
            100 : 12 : : 6
            400   8             Then, 12|00)192|00(
              8                       -----------
           ----                        Dolls. 16 Ans
  100      3200
   12         6
 ----      ----
 1200 Divisor. 19200 Dividend.
```

2. If 100 dollars gain 6 dollars in 12 months, in what time will 400 dollars gain 16?

OPERATION.

D.	*M.*	*D.*
100	: 12 : :	6
400		16
6		12
2400 *div.*		192
		100
		19200 *dividend.*

Here the blank falling under the second term, the proportion is indirect.

Therefore multiply the first, second and last terms together for a dividend, and the other two for a divisor.

M.
Then 24|00)192|00(8 Answer.
192

3. A Farmer sells 204 dolls. worth of grain in 5 years, when it is sold at 60 cents per bushel, what is it per bushel when he sells 1000 dolls. worth in 18 years, if he sell the same quantity yearly?

Cts. *Y.* *D.*
60 : 5 : : 204 *cts. m.*
18 : . 1000 : ,816 *Ans.*

4. If 7 men can reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days?

M. *D.* *A.*
7 : 12 : : 84 *M.*
5 : : 100 · 20 *Ans.*

5. If a family of 9 persons spend 450 dollars in 5 months, how much would be sufficient to maintain them 8 months, if 5 persons more were added to the family? *Ans.* $1120.

SUPPLEMENT TO THE DOUBLE RULE OF THREE.

QUESTIONS.

1. What is the Double Rule of Three; or Compound Proportion?
2. How are questions to be stated in the Double Rule of Three?
3. How is it known after the statement of the question, whether the Proportion be Direct or Inverse?
4. When the Proportion is Direct, how is the answer to be found?
5. When the Proportion is Inverse, how is the answer to be found?

EXERCISES.

1. If 6 men build a wall 20 feet long, 6 feet high and 4 feet wide in 16 days, in what time will 24 men build one 200 feet long, 8 feet high and 6 thick? *Ans.* 80 *days.*

The solid contents in each piece of wall according to the given dimensions, must be found before stating the question.

2. If 40*lb.* at Boston make 36 at Amsterdam, and 90*lb.* at Amsterdam make 116 at Dantzick, how many *lb.* at Boston are equal to 260*lb.* at Dantzick? *Ans.* $224\frac{4}{29}$*lb*

N. B. The answer to this question is found by two statements in the Rule of Three Direct.

3. If the freight of 12*Cwt.* 2*qrs.* 6*lb.* 275 miles cost \$27,78; how far may 60*Cwt.* 3*qrs.* be shipped for \$234,78? *Ans.* 480 *miles.*

4. An usurer put out 75 dollars, at interest, and at the end of eight months received for principal and interest, 79 dollars; I demand at what rate per cent he received interest? *Ans.* 8 *per cent.*

5. If 7 men can make 84 rods of wall in 6 days; in what time will 10 men make 150 rods? *Ans.* $7\frac{1}{2}$ *days.*

6. If the freight of 9*hhds.* of sugar, each weighing 12*Cwt.* 20 leagues, cost £16; what must be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$ *Cwt.* 100 leagues? *Ans.* £92 11*s.* $10\frac{2}{3}$*d*

§ 10. PRACTICE.

" PRACTICE is a contraction of the Rule of Three Direct, when the first term happens to be an unit or one ; it has its name from its daily use among Merchants and Tradesmen, being an easy and concise method of working most questions which occur in trade and business."

PROOF. By the Single Rule of Three, Compound Multiplication, or by varying the parts.

Before any advances are made in this rule, the learner must commit to memory the following

TABLES.

ALIQUOT, OR EVEN PARTS OF MONEY.

Parts of a shill.			of a £.	
d.		*s.*	and	£.
6	is	$\frac{1}{2}$	—	$\frac{1}{40}$
4	—	$\frac{1}{3}$	—	$\frac{1}{60}$
3	—	$\frac{1}{4}$	—	$\frac{1}{80}$
2	—	$\frac{1}{6}$	—	$\frac{1}{120}$
$1\frac{1}{2}$	—	$\frac{1}{8}$	—	$\frac{1}{160}$
1	—	$\frac{1}{12}$	—	$\frac{1}{240}$
$\frac{3}{4}$	—	$\frac{1}{16}$	—	$\frac{1}{320}$
$\frac{1}{2}$	—	$\frac{1}{24}$	—	$\frac{1}{480}$
$\frac{1}{4}$	—	$\frac{1}{48}$	—	$\frac{1}{960}$

5*d.* is the sum of 4*d.* & 1*d.*
7*d.* —— 6*d.* & 1*d.*
8*d.* is twice 4*d.*
9*d* is the sum of 6*d.* & 3*d.*
10*d.* —— 6*d.* & 4*d.*
11*d.* —— 6*d.* 3*d.* & 2*d.*

Pts. of a pound.			
s.	*d.*	is	£.
10	0	—	$\frac{1}{2}$
6	8	—	$\frac{1}{3}$
5	0	—	$\frac{1}{4}$
4	0	—	$\frac{1}{5}$
3	4	—	$\frac{1}{6}$
2	6	—	$\frac{1}{8}$
1	8	—	$\frac{1}{12}$
1	4	—	$\frac{1}{15}$
1	3	—	$\frac{1}{16}$
1	0	—	$\frac{1}{20}$
0	10	—	$\frac{1}{24}$
0	8	—	$\frac{1}{30}$
0	5	—	$\frac{1}{48}$
0	$2\frac{1}{2}$	—	$\frac{1}{96}$

Practice admits of a great variety of cases, the multiplicity of which serves little else than that of confounding the mind of the scholar ; a different method will be pursued here, and the whole comprised, in a few cases, such as shall be useful and easy for the scholar to bear in his memory.

The small number of examples under each case will be made up in the Supplement ; this will lead the scholar to a more particular consideration of them.

OPERATIONS.

POUNDS, SHILLINGS, PENCE, FARTH.

When the price of the given quantity is £1. 1*s.* 1*d.* per pound, yard, &c. then will the quantity itself be the answer at the supposed price.—Therefore,

CASE 1.

When the price of 1 *yard, pound, &c. consists of farthings only ;* If it be one farthing, take a fourth part of the quantity ; if a half penny, take a half ; if three farthings, take a half and a fourth of the quantity and add them. This gives the value in pence, which must be reduced to pounds.

DOLLARS, CENTS, MILLS.

RULE.

Multiply the quantity by the price of one pound, yard, &c. the product will be the answer.

POUNDS, SHILLINGS, PENCE, FARTH.

EXAMPLES.

1. What will 362 yards cost at ½d. per yard?

OPERATION.

2)362
12)181 *pence.*

15s. 1d. *Ans.*

Here the quantity stands for the price at one penny per yard, but as two farthings are but half one penny, therefore dividing the quantity by 2, gives the price at half a penny per yard, which must be reduced to shillings.

2. What will 354½ yards cost at ¼d. per yard?

OPERATION.

	d.	q.
4)	354 ½	2
12)	88	2
	7s. 4d.	2 *Ans.*

3. What will 263 yards cost at 3q. per yard? *Ans.* 16s. 5¼d.

4. What will 816 yards cost at 1q. per yard? *Ans.* 17s.

DOLLARS, CENTS, MILLS.

1. What will 362 yards cost at 7 mills per yard?

OPERATION.

3 6 2 *quantity.*
,0 0 7 *price.*

$2, 5 3 4 *Ans.*

Note. The answers in the different kinds of money will not always compare, because in the reduction of the price, a small fraction is often lost or gained.

2. What will 354½ yards cost at 3 mills per yard?

OPERATION.

3 5 4 ,5 *quantity.*
,0 0 3 *price.*

$1 ,0 6 3 5

3. What will 263 yards cost at 1 cent per yard? *Ans.* $2,63.

4. What will 816 yards cost at 3 mills per yard? *Ans.* $2,448.

POUNDS, SHILLINGS, PENCE, FARTH.

5. What will 97 yards cost at 3*q*. per yard? *Ans.* 6*s*. 0¾*d*.

6. What will 126 yards cost at ½*d*. per yard? *Ans.* 5*s*. 3*d*.

CASE 2.

When the price of 1*lb*. 1*yd*. *&c. consists of pence, or of pence and farthings;* if it be an even part of a shilling, find the value of the given quantity at 1*s*. per yard, (the quantity itself expresses the price at 1*s*. per yard; if there are quarters, &c. write for ¼ 3*d*. for ½ 6*d*. for ¾ 9*d*.) and divide by that *even part* which the price is of 1 shilling. If the price be not an aliquot or even part of one shilling, it must be divided into two or more aliquot parts; calculate for these separately, and add the values; the answer will be obtained in shillings, which must be reduced to pounds.

DOLLARS, CENTS, MILLS.

5. What will 97 yards cost at 1 cent per yard? *Ans.* ,97*cts*.

6. What will 126 yards cost at 7 mills per yard? *Ans.* $0,882.

POUNDS, SHILLINGS, PENCE, FARTH.

EXAMPLES.

1. What will 476 yards cost at $7\frac{1}{2}d.$ per yard?

OPERATION.

		s.
6d.	$\frac{1}{2}$	476 *Price at* 1s. *per yard.*
$1\frac{1}{2}d.$	$\frac{1}{4}$	238 *Price at* 6d. *per yard.*
		59 6d. *price* $1\frac{1}{2}d.$ *per yard.*

2|0)29|7 6d. *price at* $7\frac{1}{2}$ *per yd.*

£14 17s. 6d. *Ans.*

PROOF.

1. By the Rule of Three.

Y.	£.	s.	d.	Y.
As 476 :	14	17	6 : :	1

```
 20
 ---
 297
  12
 ----
476)3570(7d.
    3332
    ----
     238
       4
    ----
    )952(2qr.
     952
    ----
```

2. By Compound Multiplication.

£.	s.	d.	
		$7\frac{1}{2}$	*price of* 1 *yard.*
		10	
	6	3	*price of* 10 *yards.*
		10	
3	2	6	*price of* 100 *yards.*
		4	
12	10	0	*price of* 400 *yards.*
2	3	9	*price of* 70 *yards.*
	3	9	*price of* 6 *yards.*
£14	17	6	*price of* 476 *yards.*

DOLLARS, CENTS, MILLS.

7. What will 476 yards come to at 10 cents 4 mills per yard?

OPERATION.

```
  476
 ,104
 ----
 1904
4760
-----
$49,504 Ans.
```

PROOF.

```
cts. m.  D. cts. m.  yds.
,10  4)4 9. 5 0  4(476
       4 1 6
       -----
         7 9 0
         7 2 8
         -----
           6 2 4
           6 2 4
```

POUNDS, SHILLINGS, PENCE, FARTH.

What will 176 yards cost at $9\frac{1}{2}d.$ per yard?

OPERATION.

		s.
6d.	$\frac{1}{2}$	176 value at 1s. per yard.
3d.	$\frac{1}{2}$	88 value at 6d. per yard.
$\frac{1}{2}d.$	$\frac{1}{6}$ of	44 value at 3d. per yard.
		7 4d. val. at $\frac{1}{2}d.$ per yd.

2|0)13|9 4d.—at $9\frac{1}{2}d.$ per yd.
£6 19s. 4d. *Ans.*

PROOF.

3. What will $568\frac{3}{4}$ yards cost at 7d. per yard? *Ans.* £16 11s. $5\frac{1}{4}d$

DOLLARS, CENTS, MILLS.

8. What will 176 yards cost at 13 cents, 2 mills, per yard? *Ans.* $23,232.

9. What will $568\frac{1}{4}$ yards cost at 9 cents 7 mills per yard? *Ans.* $55,12.

POUNDS, SHILLINGS, PENCE, FARTH.

4. What will 685¾ yards come to at 2½*d.* per yard?

Ans. £7 2*s.* 10¼*d.*

5. What will 649¼ yards cost at 0*d.* per yard? *Ans.* £27 1*s.* 0½*d.*

6. What will 683¾ yards cost at 8¼*d.* per yard? *Ans.* £23 10*s.* 0¾*d.*

DOLLARS, CENTS, MILLS.

10. What will 685¾ yards come to at 3 cents 5 mills per yard?

Ans. $24,001.

11. What will 649¼ yards cost at 13 cents 9 mills per yard?

Ans. $90,245.

12. What will 683¾ yards cost at 11 cents 7 mills per yard?

Ans. 79,998

POUNDS, SHILLINGS, PENCE, FARTH. | DOLLARS, CENTS, MILLS.

CASE 3.

If the price of 1lb. 1yd. &c. be shillings and pence and an even part of £1, Divide the value of the given quantity at £1 per yard by that *even part*, which the price is of £1. The quotient will be the answer.

EXAMPLES.

1. What will 719½ yards cost at 1*s.* 4*d.* per yard?

OPERATION.

		£.	s.	
1*s.* 4*d.*	$\frac{1}{15}$	719	10	price at £1 per yd.
		143	18	price at 4*s.* per yd.
		Ans. 47	19 4*d.*	at 1*s.* 4*d.* per yd.

Here for the sake of ease in the operation, because 5×3=15, therefore I divide the price at one pound per yard by 5, and that quotient by 3 which gives the answer.

13. What will 719½ yards cost at 22 cents, 3 mills per yard? *Ans.* $160,448.

2. What will 648 yards cost at 1*s.* 8*d.* per yard? *Ans.* £54.

14. What will 648 yards cost at 27 cents and 8 mills per yard? *Ans.* $180,144.

3. What will 167½ yards cost at 3*s.* 4*d.* per yard? *Ans.* £27 18*s.* 4*d.*

15. What will 167½ yards cost at 55 cents, 6 mills per yard? *Ans.* $93,13.

POUNDS, SHILLINGS, PENCE, FARTH.

4. What will $687\frac{1}{2}$ yards cost at 5*s.* per yard? *Ans.* £171 17*s.* 6*d.*

CASE 4.

When the price of 1*yd. &c. is shillings, or shillings, pence and farthings, and not an even part of* £1. Multiply the value of the quantity at 1*s.* per yard by the number of shillings; for the pence and farthings, take parts, as in case 2, the results added will give the answer, which must be reduced to pounds.

If the price be shillings only, and an even number; multiply by half the price or even number of shillings for one yard, double the unit figure of the product for shillings, the remaining figure will be pounds.

NOTE. When the quantity contains a fraction, work for the integers, and for the fraction take proportional parts of the rate.

EXAMPLES.

1. What will $167\frac{1}{2}$ yards cost at 17*s.* 6*d.* per yard?

OPERATION.

```
               s.
| 6d. | ½ |   167
               17
              ----
              1169
              167
              ----
              2839 price at 17s. per yd.
                83  6—at 6d. per yard.
                 8  9 price of ½ yard.
              ----
         2|0)293|1  3d.

   Ans. £146 11s. 3d.
```

DOLLARS, CENTS, MILLS.

16. What will $687\frac{1}{2}$ yards cost at 83 cents, 3 mills per yard? *Ans.* \$572,687.

17. What will $167\frac{1}{2}$ yards cost at \$2,916? *Ans.* \$488,43.

POUNDS, SHILLINGS, PENCE, FARTH.

2. What will 5482 yards cost at 12*s.* 4½*d.* per yard?
Ans. £3391 19*s.* 9*d.*

3. What will 614 yards cost at 16*s.* per yard?

OPERATION.

614
8 *half the price.*

4912 *double the first figure*
£491 4*s.* *Ans.* [*for shill.*

4. What will 176 yards cost at 12*s.* per yard? *Ans.* £105 12*s.*

5. What will 36 yards cost at 7*s.* 6*d.* per yard? *Ans.* £13 10*s*

DOLLARS, CENTS, MILLS.

18. What will 5482 yards cost at $2,063 per yard?
Ans. $11309,366

19. What will 614 yards cost at $2,667 per yard?
Ans. $1637,538.

20. What will 176 yards cost at $2 per yard? *Ans.* $352.

21. What will 36 yards cost at $1,25 per yard? *Ans.* $45.

POUNDS, SHILLINGS, PENCE, FARTH.

CASE 5.

When the price of 1yd. 1lb. &c. is pounds, shillings and pence; multiply the quantity by the pounds, and if the shillings and pence be an even part of a pound, divide the given quantity by that *even part*, and add the quotient to the product for the answer: but if they are not an even part of £1, take parts of parts and add them together. Or, you may reduce the pound in the price of 1 yard, &c. to shillings, and proceed as in the case before.

EXAMPLES.

1. What will 59 yards cost, at £6 7*s*. 6*d*. per yard?

OPERATION.

	£.
5*s*. is $\frac{1}{4}$ of £1.	59 *value of £1 per yd.*
	6
	354—*at £6 per yd.*
2*s*. 6*d*. is $\frac{1}{2}$ of 5.	14 15*s*. *at 5s. per yd.*
	7 7 6*d*. *at 2s. 6d. per yd.*

Ans. £376 2*s*. 6*d*. *at £6 7s. 9d.*

3. What will 163 yards cost, at £2 8*s*. per yard? *Ans*. £391 4*s*.

DOLLARS, CENTS, MILLS

22. What will 59 yards cost, at \$21,25 per yard?

OPERATION.

```
 D. C.
 21,25
    59
 ------
 191 25
1062 5
 ------
$1253 75 Ans.
```

23. What will 163 yards cost, at \$8 per yard? *Ans*. \$1304.

POUNDS, SHILLINGS, PENCE, FARTH.

3. What will 76 yards cost at £3 2*s.* 7*d.* per yard?

OPERATION.

```
                    s.
6d. is 1/2 of 1s.   76 value at 1s. per yd.
                    62=shillings in £3 2s.
                   ----
                   152 value at 2s. per yd.
                  456  —at 60s. per yd.
1d. is 1/6 of 6d.  38—at 6d. per yard.
                    6' 4d.—at 1d. per yd.
                  ------
             2|0)475|6
                  ------
Ans. £237 16s. 4d.
```

4. What is the value of 84 yards at £2 14*s.* per yard?

Ans. £226 16*s.*

DOLLARS, CENTS, MILLS.

24. What will 76 yards cost at $10,43 per yard?

Ans. $792,68.

25. What is the value of 84 yards at $9 per yard? *Ans.* $756.

SUPPLEMENT TO PRACTICE.

QUESTIONS.

1. What is Practice?
2. Why is it so called?
3. When the price of 1 yard, &c. is farthings, how is the value of any given quantity found at the same rate?
4. When the price consists of pence and farthings, and is an even part of 1*s.* how is the value of any given quantity found?
5. When the price is pence and farthings and not an even part of 1*s* what is the method of procedure?
6. When the price consists of shillings, pence and farthings, how is the value of any given quantity found?
7. When the price contains shillings and pence and an even part of £1, how is the operation to be conducted?
8. When the price consists of shillings only, and an even number, what is the most direct way to find the value of any given quantity?
9. When the quantity contains fractions, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, &c. how are they to be treated?
10. When the price consists of pounds, and lower denominations, how is the value of any given quantity found?
11. When the prices are given in dollars, cents, and mills, how is the value of any given quantity found in Federal Money?
12. What is the method of proof?
13. How are operations in Federal Money proved?

EXERCISES IN PRACTICE.

In the following exercises the attention of the scholar must be excited first to consider to which of the preceding cases each question is to be referred. That being ascertained, he will proceed in the operation according to the instruction there given.

1. What will 745$\frac{3}{4}$ yards cost at 11*d.* per yard? *Ans.* £34 3*s.* 7$\frac{1}{4}$*d.*

Under which of the preceding cases does this question properly belong?

What must be done with the fraction ($\frac{3}{4}$ of a *yard*) in the quantity?

2. What will 964 yards cost at 1*s*. 8*d*. per yard? *Ans*. £80 6*s* 8*d*.

OPERATION PROOF.

3. What will 354¼ yards cost, at ¼*d*. per yard? *Ans*. 7*s*. 4½*d*.

4. What will 316 yards cost, at ¾*d*. per yard? *Ans*. 19*s*. 9*d*.

5. What will 567½ yards cost, at 1½*d*. per yard? *Ans*. £3 10*s*. 11¼*d*.

6. What will 913½ yards cost, at 6*d*. per yard? *Ans*. £22 16*s*. 9*d*.

7. What will 912¼ yards cost, at 9*d.* per yard?

Ans. £34 4*s.* 4¼*d.*

8. What will 76 yards cost, at 2*d.* per yard?

Ans. 12*s.* 8*d.*

9. What will 845 yards cost, at 8*s.* per yard?

Ans. £338.

10. What will 91 yards come to at 16*s.* per yard?

Ans. £72 16*s.*

11. What will 156½ yards come to, at 6*s.* 4*d.* per yard?

Ans. £49 11*s.* 2*d.*

12. What will 96 yards cost at 10*s.* 1½*d.* per yard?

Ans. £48 12*s.*

13. What will 67½ yards cost, at 12*s.* 2*d.* per yard? *Ans.* £41 1*s.* 3*d.*

14. What will 843 yards cost, at 6*s* 8*d.* per yard? *Ans.* £281.

15. What will 75 yards cost, at £3 3*s.* 4*d.* per yard?
Ans. £237 10*s.*

16. What will 59 yards come to, at £6 7*s.* 6*d.* per yard?
Ans. £376 2*s.* 6*d.*

17. What will 59¾ yards come to, at £3 6*s.* 8*d.* per yard?
Ans. £199 3*s.* 4*d.*

18. What will 68 yards cost, at £4 6*s.* per yard?
Ans. £292 8*s.*

N. B. The following questions are left without any answers, that the Scholar may operate and prove each question.

19. What will 11 yards of flannel, at 2s. 6d. per yard come to?

OPERATION. PROOF.

20. What will 13lb. of cotton cost at 3s. 4d. per lb.?

21. What will 183 yards of ribbon come to at 8d. per yard?

THE

SCHOLAR'S ARITHMETIC

SECTION III.

RULES OCCASIONALLY USEFUL TO MEN IN PARTICULAR CALLINGS AND PURSUITS OF LIFE.

§ 1. INVOLUTION.

INVOLUTION, or the raising of powers, is the multiplying of any given number into itself continually, a certain number of times. The quantities in this way produced, are called powers of the given number. Thus,

$4\times4=16$ is the second power or square of 4. $=4^2$

$4\times4\times4=64$ is the 3d power, or cube of 4. $=4^3$

$4\times4\times4\times4=256$ is the 4th power or biquadrate of 4. $=4^4$

The given number, (4) is called the first power; and the small figure, which points out the order of the power, is called the *Index* or the *Exponent*.

§ 2. EVOLUTION.

EVOLUTION, or the extraction of roots, is the operation by which we find any root of any given number.

The root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d, 3d, 4th, root, &c. accordingly as it is, when raised to the 2d, 3d, 4th, &c power, equal to that power. Thus, 4 is the square root of 16, because $4\times4=16$. 4 also is the cube root of 64, because $4\times4\times4=64$; and 3 is the square root of 9, and 12 is the square root of 144, and the cube root of 1728, because $12\times12\times12=1728$, and so on.

To every number there is a root, although there are numbers, the precise roots of which can never be obtained. But by the help of decimals, we can approximate towards those roots, to any necessary degree of exactness. Such roots are called *Surd Roots*, in distinction from those perfectly accurate, which are called *Rational Roots*.

The square root is denoted by this character $\sqrt{}$ placed before the power; the other roots by the same character, with the index of the root placed over it. Thus the square root of 16 is expressed $\sqrt{16}$, and the cube root of 27 is $\sqrt[3]{27}$, &c.

When the power is expressed by several numbers, with the sign + or − between them, a line is drawn from the top of the sign over all the parts of it; thus the second power of 21—5 is $\sqrt{21-5}$, and the 3d power of 56+8 is $\sqrt[3]{56+8}$, &c.

The second, third, fourth and fifth powers of the nine digits may be seen in the following

TABLE.

Roots - - - -	or 1st Powers	1	2	3	4	5	6	7	8	9
Squares - - -	or 2d Powers	1	4	9	16	25	36	49	64	81
Cubes - - - -	or 3d Powers	1	8	27	64	125	216	343	512	729
Biquadrates	or 4th Powers	1	16	81	256	625	1296	2401	4096	6561
Sursolids - -	or 5th Powers	1	32	243	1024	3125	7776	16807	32768	59049

§ 3. EXTRACTION OF THE SQUARE ROOT.

To extract the square root of any number, is to find another number which multiplied by or into itself, would produce the given number; and after the root is found, such a multiplication is a proof of the work.

RULE.

1. " Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. " Find the greatest square number in the first, or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. " Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. " Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it; and likewise on the right hand of the divisor; multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend; to the remainder join the next period for a new dividend.

5. "Double the figures already found in the root for a new divisor, (or bring down your last divisor for a new one, doubling the right hand figure of it) and from these find the next figure in the root, as last directed, and continue the operation in the same manner till you have brought down all the periods."

"*Note* 1. If, when the given power is pointed off as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period."

"*Note* 2. If there be decimals in the given number it must be pointed both ways from the place of units; If, when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many cyphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing cyphers."

EXAMPLES.

1. What is the square root of 729?

OPERATION.

```
     7̇2̇9(27 the root.
     4
    ———
 47)329
    329
    ———
    000

PROOF.
     27
     27
    ———
    189
    54
    ———
    729
```

The given number being distinguished into periods, I seek the greatest square number in the left hand period (7) which is 4, of which the root (2) being placed to the right hand of the given number, after the manner of a quotient, and the square number (4) subtracted from the period (7) to the remainder (3) I bring down the next period (29) making for a dividend, 329. Then the double of the root (4) being placed to the left hand for a divisor, I say how often 4 in 32? (*excepting* 9 *the right hand figure*) the answer is 7, which I place in the root for the second figure of it, and also to the right hand of the divisor; then multiplying the divisor thus increased by the figure (7) last obtained in the root, I place the product underneath the dividend, and subtract it therefrom, and the work is done.

DEMONSTRATION

Of the reason and nature of the various steps in the extraction of the SQUARE ROOT.

The superficial content of any thing, that is the number of square feet, yards or inches, &c. contained in the surface of a thing, as of a table or floor, a picture, a field, &c. is found by multiplying the length into the breadth. If the length and breadth be equal, it is a square, then the measure of one of the sides as of a room, is the root, of which the superficial content in the floor of that room is the second power. So that having the superficial contents of the floor of a square room, if we extract the square root, we shall have the length of one side of that room. On the other hand, having the length of one side of a square room, if we multiply that number into itself, that is, raise it to the second power, we shall then have the superficial contents of the floor of that room.

The extraction of the square root therefore has this operation on numbers, *to arrange the numbers of which we extract the root into a square*

form. As if a man should have 625 yards of carpeting 1 yard wide, if he extract the square root of that number (625) he will then have the length of one side of a square room, the floor of which, 625 yards will be just sufficient to cover.

To proceed then to the demonstration.

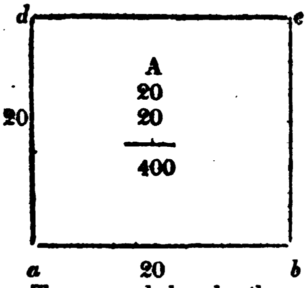
EXAMPLE 2. Supposing a man has 625 yards of carpeting, 1 yard wide, what will be the length of one side of a square room, the floor of which his carpeting will cover.

The first step is to point off the number into periods of two figures each. This determines the number of figures of which the root will consist, and is done on this principle, *that the product of any two numbers can have at most but so many places of figures as there are places in both the factors, and at least but one less, of which any person may satisfy himself at pleasure.*

OPERATION.

$$\begin{array}{l} \dot{6}2\dot{5}(20 \\ 4 \\ \hline 225 \end{array}$$

FIG. I.



The number being pointed off as the rule directs, we find we have two periods, consequently the root will consist of two figures. The greatest square number in the left hand period (6) is 4, of which two is the root, therefore, 2 is the first figure of the root, and as it is certain we have one figure more to find in the root, we may for the present supply the place of that figure by a cypher (20) then 20 will express the just value of that part of the root now obtained. But it must be remembered, that a root is the side of a square of equal sides. Let us then form a square. A, *Fig. I.* each side of which shall be supposed 20 yards. Now the side *a b* of this square or either of the sides, shews the root 20, which we have obtained.

To proceed then by the rule "*Place the square number underneath the period, subtract, and to the remainder bring down the next period.*"—Now the square number (4) is the superficial contents of the square A made evident thus—each side of the square A measures 20 yards, which number multiplied into itself, produces 400, the superficial contents of the square A, also the square number, or the square of the figure 2 already found in the root, is 4, which placed under the period (6) as it falls in the place of hundreds, is in reality 400, as might be seen also by filling the places to the right hand with cyphers, then 4 subtracted from 6 and to the remainder (2) the next period (25) being brought down, it is plain, the sum 625 has been diminished by the reduction of 400, a number equal to the superficial contents of the square A.

Hence *Fig. I.* exhibits the exact progress of the operation. By the operation 400 yards of the carpeting have been disposed of, and by the figure is seen the disposition made of them.

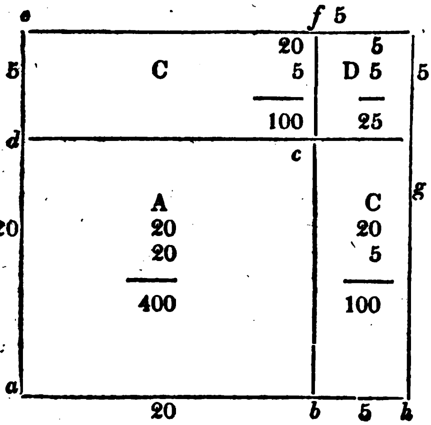
Now the square A is to be enlarged by the addition of the 225 yards which remain, and this addition must be so made that the figure at the same time shall continue to be a complete and perfect square. If the addition be made to one side only, the figure would lose its square form, it must be made to two sides; for this reason the rule directs, "place the double of the root already found on the left hand of the dividend for a divisor." The double of the root is just equal to two sides *b c* and *c d* of the square, A, as may be seen by what follows

OPERATION *continued*:

```
 625(25
 4
 ———
45)225
   225
   ———
```

The double of the root is 4, which placed for a divisor in place of tens (*for it must be remembered that the next figure in the root is to be placed before it*) is in reality 40, equal to the sides *b c* (20) and *c d* (20) of the square A.

Fig. II.



Again, by the rule, "seek how often the divisor is contained in the dividend (except the right hand figure) and place the answer in the root. for the second figure of it, and on the right hand of the divisor."

Now if the sides *b c* and *c d* of the square A Fig. II. is the length to which the remainder 225 yds. are to be added, and the divisor (4 tens) is the sum of these two sides, it is then evident that 225 divided by the length of the two sides, that is by the divisor (4 tens) will give the breadth of this new addition of the 225 yards to the sides *b c* and *c d* of the square A.

The square A	=400	yards.
—— C *e f*	=100	—
—— C *g h*	=100	—
—— D	= 25	—
Proof	625	*yards.*

But we are directed to "*except the right hand figure*," and also to "*place the quotient figure on the right hand of the divisor*;" the reason of which is that the addition, C *e f* and C *g h* to the sides *b c* and *c d* of the square, A, do not leave the figure a complete square, but there is a deficiency D, at the corner.—Therefore in dividing the right hand figure is excepted, to leave something of the dividend, for this deficiency; and as the deficiency D, is limited by the additions C *e f* and C *g h*, and as the quotient figure (5) is the width of these additions, consequently equal to one side of the square D; therefore the quotient figure (5) placed to the right hand of the divisor (4 tens) and multiplied into itself, gives the contents of the square D, and the 4 tens=to the sum of the sides, *b c* and *c d* of the addition of C *e f* and C *g h*, multiplied by the quotient figure (5) the width of those additions give the contents C *e f* and C *g h*, which together subtracted from the dividend, and there being no remainder, shew that the 225 yards are disposed in these new additions C *e f*, C *g h*, and D, and the figure is seen to be continued a complete square.

Consequently, *Fig. II.* shews the dimensions of a square room, 25 yards on a side, the floor of which 625 yards of carpeting, 1 yard wide will be sufficient to cover.

The *Proof* is seen by adding together the different parts of the figure.

Such are the principles on which the operation of extracting the square root is grounded.

3. What is the square root of 10342656? *Ans.* 3216.

4. What is the square root of 43264? *Ans.* 208.

5. What is the square root of 964,5192360241? *Ans.* 31,05671.

6. What is the square root of 998001? *Ans.* 999.

7. What is the square root of 234,09? *Ans.* 15,3.

8. What is the square root of 1030892198,4001? *Ans.* 32107,51.

SUPPLEMENT TO THE SQUARE ROOT.

QUESTIONS.

1. What is to be understood by a root? A power? The second, third and fourth powers?
2. What is the Index, or Exponent?
3. What is it to extract the Square Root?
4. Why is the given sum pointed into periods of two figures each?
5. In the operation, having found the first figure in the root, why do we subtract the square number, that is, the square of that figure from the period in which it was taken?
6. Why do we double the root for a divisor?
7. In dividing why do we except the right hand figure of the dividend?
8. Why do we place the quotient figure in the root, and also to the right hand of the divisor?
9. If there be decimals in the given number how must it be pointed?
10. How is the operation of extracting the Square Root proved?

EXERCISES IN THE SQUARE ROOT.

1. A Clergyman's glebe consists of three fields: the first contains 5 *acres*, 2*r*. 12*p*. the second, 2 *acres*, 2*r*. 15*p*. the third, 1 *acre*, 1*r*. 14*p*. in exchange for which the heritors agree to give him a square field equal to all the three. Sought the side of the square? *Ans.* 39 *poles.*

2. A general has an army of 4096 men; how many must he place in rank and file to form them into a square? *Ans.* 64

3. There is a circle whose diameter is 4 inches; what is the diameter of a circle 4 times as large? *Ans.* 8 inches.

NOTE. Square the given diameter, multiply this square by the given proportion, and the square root of the product will be the diameter required. Do the same in all similar cases.

If the circle of the required diameter were to be less than the circle of the given diameter, by a certain proportion, then the square of the given diameter must have been divided by that proportion.

4. There are two circular ponds in a gentleman's pleasure ground; the diameter of the less is 100 feet, and the greater is three times as large.—What is its diameter? *Ans.* 173,2+

5. If the diameter of a circle be 12 inches, what will be the diameter of another circle half so large? *Ans.* 8,48+*inches.*

6. A wall is 36 feet high, and a ditch before it is 27 feet wide; what is the length of a ladder, that will reach to the top of the wall from the opposite side of the ditch? *Ans.* 45 *feet.*

NOTE. A Figure of three sides, like that formed by the wall, the ditch and the ladder, is called a *right angled triangle*, of which the square of the hypothenuse, or slanting side, (*the ladder*) is equal to the sum of the squares of the two other sides, that is, the height of the wall and the width of the ditch.

7. A line of 36 yards will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad; the height of the wall is required? *Ans.* 26,83+ *yards.*

8. Glasgow is 44 miles west from Edinburgh; Peebles is exactly south from Edinburgh, and 49 miles in a straight line from Glasgow; what is the distance between Edinburgh and Peebles? *Ans.* 21,5+*miles*

§ 4. EXTRACTION OF THE CUBE ROOT.

To extract the Cube Root of any number is to find another number, which multiplied into its square shall produce the given number.

RULE.

1. " Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. " Find the greatest cube in the left hand period, and put its root in the quotient.

3. " Subtract the cube thus found, from the said period, and to the remainder bring down the next period, and call this the dividend.

4. " Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor*.

5. " Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. " Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure, and call their sum the *subtrahend*.

7. " Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.

NOTE. The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root."

1. What is the cube root of 373248?

OPERATION.

373248(72 *the root.*	
343	7×7×300=14700, *the triple square.*
	7×30 = 210 *the triple quotient*
Divisor 14910)30248	
	14910 *the divisor.*
29400	14700×2=29400
840	2×2×210= 840
8	2×2×2 = 8
30248	30248 *the subtrahend.*
00000	

DEMONSTRATION

Of the reason and nature of the various steps in the operation of extracting the Cube Root.

Any solid body having *six equal sides*, and each of the sides an *exact square* is a Cube, and the measure in length of one of its sides is the root of that cube. For if the measure in feet of any one side of such a body be multiplied three times into itself, that is, raised to the third power, the product will be the number of solid feet the whole body contains.

And on the other hand, if the cube root of any number of feet be extracted, this root will be the length of one side of a cubic body, the whole contents of which will be equal to such a number of feet.

Supposing a man has 13824 feet of timber, in distinct and separate blocks of one foot each; he wishes to know how large a solid body they will make when laid together, or what will be the length of one of the sides of that cubic body?

To know this, all that is necessary is to extract the cube root of that number, in doing which I propose to illustrate the operation.

OPERATION.

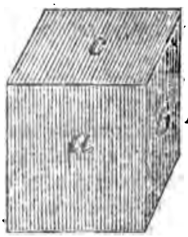
13824(20
8

5824

In this number, pointed off as the rule directs, there are two periods: of course there will be two figures in the root.

The greatest cube in the right hand period (13) is 8, of which 2 is the root, therefore 2 placed in the quotient is the first figure of the root, and as it is certain we have one figure more to find in the root, we may for the present supply the place of that one figure by a cypher (20) then 20 will express the true value of that part of the root now obtained. But it must be remembered, that the cube root is the length of one of the sides of the cubic body, whose length, breadth and thickness are equal. Let us then form a cube, *Fig.* 1, each side of which shall be supposed 20 feet; now the side A B of this cube, or either of the sides, shews the root (20) which we have obtained.

Fig. I.



20
400
20

8000 feet=the solid contents of the Cube.

The Rule next directs, *subtract the cube thus found from the said period, and to the remainder bring down the next period, &c.* Now this cube (8) is the solid contents of the figure we have in representation. Made evident thus.—Each side of this figure is 20, which being raised to the 3d power, that is the length, breadth and thickness being multiplied into each other, gives the solid contents of that figure=8000 feet, and the cube of the root, (2) which we have obtained, is 8, which placed under the period from which it was taken as it falls in the place of thousands, is 8000, equal to the solid contents of the cube A B C D E F, which being subtracted from the given number of feet, leaves 5824 feet.

Hence, *Fig. I.* exhibits the exact progress of the operation. By the operation 8000 feet of the timber are disposed of, and the figure shews the disposition made of them, into a solid pile, which measures 20 feet on every side.

Now this figure or pile is to be enlarged by the addition of the 5824 feet, which remains, and this addition must be so made, that the figure or pile shall continue to be a complete cube, that is, have the measure of all its sides equal.

To do this the addition must be made equally to the three different squares, or faces *a*, *c* and *b*.

The next step in the operation is, to find a divisor; and the proper divisor will be, the number of square feet contained in all the points of the figure, to which the addition of the 5824 feet is to be made.

Hence we are directed to "*multiply the square of the quotient by* 300," the object of which is to find the superficial contents of three faces *a*, *c*, *b*, to which the addition is now to be made. And that the square of the quotient, multiplied by 300 gives the superficial contents of the faces *a*, *c*, *b*, is evident from what follows:

Side A B=20	*of the face a.*	2 *quotient figure.*
Side A F=20		2
Superficial contents=400		4 *the square of* 2.
3		300

The triple square 1200=*the superficial contents of the faces a, c, and b.*

The two sides A B & A F of the face *a*, multiplied into each other, give the superficial content of *a*, and as the faces, *a*, *c*, and *b*, are all equal, therefore the content of face *a*, multiplied by 3, will give the contents of *a*, *c*, and *b*.

The triple square 1200=*the superficial contents of the faces a, c, and b.*

Here the quotient figure 2 is properly, *two tens*, for there is another figure to follow it in the root, and the square of 2, standing as units, is 4, but its true value is 20 (*the side A B*) of which the square is 400, we therefore lose two cyphers, and these two cyphers are annexed to the figure 3—Hence it appears that we square the quotient with a view to find the superficial content of the face or square *a*, we multiply the square of the quotient by 3, to find the superficial contents of the three squares, *a*, *c*, and *b*, and two cyphers are annexed to the 3, because in the square of the quotient *two cyphers* were lost, the quotient requiring a cypher before it in order to express its true value, which would throw the quotient (2) into the place of *tens*, whereas now it stands in the place of units.

Now when the additions are made to the squares *a*, *c*, and *b*, there will evidently be a deficiency, along the whole length of the sides of the squares, between each of the additions, which must be supplied before the figure can be a complete cube. These deficiencies will be 3, as may be seen, *Fig. II. n n n.*

Therefore it is, that we are directed, "*multiply the quotient by 30, calling it the triple quotient.*"

The triple quotient is the sum of the three lines, or sides, against which are the deficiencies *n n n*, all which meet at a point, nigh the centre of the figure. This is evident from what follows.

The deficiencies are three in number, they are the whole length of the sides, the length of each side is 20 feet,

Therefore 20
3
—
Triple quotient 60=*to the length of* 3 *sides where are deficiencies to be filled.*

2 quotient.
30
—
Triple quotient 60 *equal the length of* 3 *sides, &c.*

Here as before, the quotient lacks a cypher to the right hand, to exhibit its true value; the quotient itself is the length of one of the sides, where are the deficiencies; it is multiplied by 3 because there are 3 deficiencies, and a cypher is annexed to the 3 because it has been omitted in the quotient, which gives the same product, as if the true value of the quotient 20, had been multiplied by 3 alone.

We now have { 1200 the triple square.
60 the triple quotient.
——

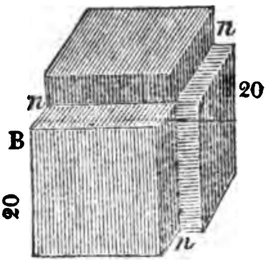
The sum of which, 1260 is the divisor, equal the number of square feet contained in all the points of the figure or pile, to which the addition of the 5824 feet is to be made.

OPERATION, *continued.*

13824(24 *the root.*
8
——
Divis. 1260)5824 *the dividend.*
4800
960
64
——
5824 *subtrahend.*
——
0000

This Figure in the root (4) shews the depth of the addition on every point where it is to be made to the pile or figure, represented, *Fig. I.*

FIG. II.
20



A 20 F

FIG. II. exhibits the additions made to the squares *a c b*, by which they are covered or raised by a depth of 4 feet.

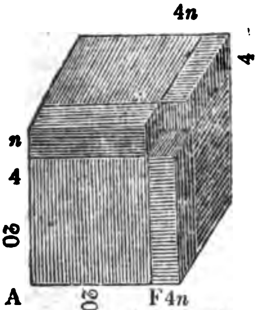
The next step in the operation is to find a subtrahend, which subtrahend is the number of solid feet contained in all the additions to the cube, by the last figure 4.

Therefore the rule directs, *multiply the triple square by the last quotient figure.*

The triple square it must be remembered, is the superficial contents of the faces *a c* and *b*, which multiplied by 4, the depth now added to these faces, or squares, gives the number of solid feet contained in the additions by the last quotient figure 4.

1200 *triple square.*
4 *last quotient figure.*
——
4800 *feet, equal the addition made to the squares, or faces, a, c, b, of Fig. I a depth of* 4 *feet on each.*

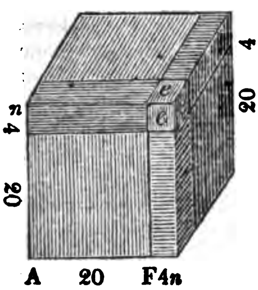
FIG. III.



Then, "*Multiply the square of the last quotient figure by the triple quotient.*" This is to fill the deficiencies, *n n n*, *Fig. II.* Now these deficiencies are limited in length by the length of the sides (20) and the triple quotient is the sum of the length of the deficiencies. They are limited in width by the last quotient figure (4) the square of which gives the area or superficial contents at one end, which multiplied into their length, or the triple quotient, which is the same thing, gives the contents of those additions 4*n*4, 4*n*, 4*n*, *Fig. III.*

60 *triple quotient.*
16 *square of the last quotient.*
———
360
60
———
960 *feet disposed in the deficiencies, between the additions to the squares a c b, Fig. III. exhibits these deficiencies, supplied 4n4, 4n, 4n, and discovers another deficiency where these approach together, of a corner wanting to make the figure a complete cube.*

FIG. IV.



Lastly, "*Cube the last quotient figure.*" This is done to fill the deficiency, *Fig. III.* left at one corner, in filling up the other deficiencies, *n n n*. This corner is limited by those deficiencies on every side, which were 4 feet in breadth, consequently the square of 4 will be the solid content of the corner which in *Fig. IV. e e e is seen filled.*

Now the sum of these additions make the subtrahend, which subtract from the dividend and the work is done.

4
4
———
16
4
———
64 *feet is the corner e e e, where the additions n n n, approach together.*

Figure IV. Shews the pile which 13824 solid blocks of one foot each, would make when laid together. The root (24) shews the length of a side. Fig. I. shews the pile which would be formed by 8000 of those blocks, first laid together; Fig. II. Fig. III. and Fig. IV. shew the changes which the pile passes through in the addition of the remaining 5824 blocks or feet.

Proof. By adding the contents of the first figure, and the additions exhibited in the other figures together.

Feet.
8000 Contents of *Fig. I.*
4800 addition to the faces or square *a*, *c*, and *b*, *Fig. II.*
960 addition to fill the deficiencies *n*, *n*, *n*, *Fig. III.*
64 addition at the corner, *e*, *e*, *e*, *Fig. IV* where the additions
——— which fill the deficiencies *n*, *n*, *n*, approach together.
13824 Number of blocks or solid feet, all which are now disposed in Fig. IV. forming a pile or solid body of timber, 24 feet on a side.

Such is the demonstration of the reason and nature of the various steps in the operation of extracting the cube root. Proper views of the figures, and of those steps in the operation illustrated by them, will not generally be acquired without some diligence or attention. Scholars more especially will meet with difficulty. For their assistance, small blocks might be formed of wood in imitation of the Figures, with their parts in different pieces. By the help of these, Masters, in most instances, would be able to lead their pupils into the right conceptions of those views, which are here given of the nature of this operation.

3. What is the cube root of 21024576? *Ans.* 276.

4. What is the cube root of 253395799552? *Ans.* 6328.

5. What is the cube root of 84,604519 ? *Ans.* 4,39.

6. What is the cube root of 2 ? *Ans.* 1,25+

SUPPLEMENT TO THE CUBE ROOT.

QUESTIONS.

1. What is a Cube?
2. What is understood by the cube root?
3. What is it to extract the cube root?
4. In the operation, having found the first figure of the root, why is the cube of it subtracted from the period in which it was taken?
5. Why is the square of the quotient multiplied by 300?
6. Why is the quotient multiplied by 30?
7. Why do we add the triple square and triple quotient together, and the sum of them call the divisor?
8. To find the subtrahend, why do we multiply the triple square by the last quotient figure? the square of the last quotient figure by the triple quotient? Why do we cube the quotient figure? Why do these sums added, make the subtrahend?
9. How is the operation proved?

EXERCISES IN THE CUBE ROOT.

1. If a bullet 6 inches in diameter weigh 32lb. what will a bullet of the same metal weigh whose diameter is 3 inches? *Ans.* 4lb.

NOTE. "The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides, or diameters."

2. What is the side of a cubical mound, equal to one 288 feet long, 216 broad and 48 feet high? *Ans.* 144 *feet.*

3. There is a cubical vessel whose side is two feet; I demand the side of a vessel which shall contain three times as much?

Ans. 2 *feet ten inches and* $\frac{3}{5}$ *nearly.*

Note. Cube the given side, multiply it by the given proportion, and the cube root of the product will be the side sought.

§ 5. FELLOWSHIP.

FELLOWSHIP is a rule by which merchants and others, trading in partnership, compute their particular shares of the gain or loss, in proportion to their stock and the time of its continuance in trade.

It is of two kinds, *Single* and *Double*.

SINGLE FELLOWSHIP,

Is when the stocks are employed equal times.

RULE.

As the whole sum of the stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain or loss.

PROOF. Add all the shares of the gain or loss together; and if the work be right, the sum will be equal to the whole gain or loss.

EXAMPLES.

1. Two merchants, A and B, make a joint stock of 200 dollars; A puts in 75 dollars, and B 125 dollars; they trade and gain 50 dollars; what is each man's share of the gain?

OPERATION.

Dolls. Dolls. Dolls.	
As 200 : 50 : : 75	*As* 200 : 50 : : 125
75	125
250	250
350	100
D. cts.	50
200)3750(18,75 A's share.	*D. cts.*
200	200)6250(31,25 B's share.
1750	600
1600	250
1500	200
1400	500
1000 18,75 A's share.	400
1000 31,25 B's share.	1000
50,00 *Proof.*	1000

2. Divide the number 360 into 4 such parts, which shall be to each other as 3, 4, 5, and 6.

$$\left.\begin{array}{r}60\\80\\100\\120\end{array}\right\}\textit{Answer.}$$

360 *Proof.*

3. A man died leaving 3 sons, to whom he bequeathed his estate in the following manner, viz. to the eldest he gave 184 dollars, to the second 155 dollars, and to the third 96 dollars; but when his debts were paid, there were but 184 dollars left; What is each one's proportion of his estate?

Ans. 77,829
65,563 } *Shares.*
40,606

4. A and B companied A put in £45, and took $\frac{3}{5}$ of the gain; What did B put in? *Ans.* £30.

DOUBLE FELLOWSHIP.

DOUBLE FELLOWSHIP, or Fellowship with time, is when the stocks of partners are continued unequal times.

RULE.

Multiply each man's stock by the time it was continued in trade. Then, As the whole sum of the products is to the whole gain or loss, so is each man's particular product to his particular share of the loss or gain.

EXAMPLES.

1. A, B, and C, entered into partnership; A put in 85 dollars for 8 months; B put in 60 dollars for 10 months; and C put in 120 dollars for 3 months; by misfortune they lost 41 dollars: What must each man sustain of the loss?

OPERATION.

85	60	120
8	10	3
680	600	360

680 A's product.
600 B's product.
360 C's product.
1640

As 1640 : 41 : : 680
680
680
2720
164|0)2788|0(17 A's loss.
164
1148
1148
0000

As 1640 : 41 : : 600
600
164|0)2460|0(15 B's loss.
164
820
820

As 1640 : 41 : : 360
360
2460
123
164|0)1476|0(9 C's loss.
1476
0000

Dolls.
17 A's loss.
15 B's loss.
9 C's loss.
41 *Proof.*

2. A, B, and C, trade together; A, at first put in 480 dollars for 8 months, then put in 200 dollars more and continued the whole in trade 8 months longer, at the end of which he took out his whole stock; B put in 800 dollars for 9 months, then took out $583,333 and continued the rest in trade 3 months; C put in $366,666 for ten months, then put in 250 dollars more, and continued the whole in trade 6 months longer. At the end of their partnership they had cleared 1000 dollars; what is each man's share of the gain?

Ans. $378,827 A's share.
320,452 B's share.
300,721 C's share.

SUPPLEMENT TO FELLOWSHIP.

QUESTIONS.

1. What is Fellowship ?
2. Of how many kinds is Fellowship ?
3. What is Single Fellowship ?
4. What is the rule for operating in Single Fellowship ?
5. What is Double Fellowship ?
6. What is the rule for operating in Double Fellowship ?
7. How is Fellowship proved ?

EXERCISES IN FELLOWSHIP.

A, B, and C, hold a pasture in common, for which they pay £20 per annum. In this pasture, A had 40 oxen for 76 days; B had 36 oxen for 50 days, and C had 50 oxen for 90 days. I demand what part each of these tenants ought to pay of the £20.

	£	s.	d.	q.	
Ans.	6	10	2	$1\frac{2340}{9340}$	A's part.
	3	17	1	$0\frac{2000}{9340}$	B's part.
	9	12	8	$2\frac{5000}{9340}$	C's part.

§ 6. BARTER.

BARTER is the exchanging of one commodity for another, and teaches merchants so to proportion their quantities, that neither shall sustain loss.

PROOF. By changing the order of the question.

RULE.

1. *When the quantity of one commodity is given with its value or the value of its integer, as also the value of the integer of some other commodity to be exchanged for it, to find the quantity of this commodity:*—Find the value of the commodity of which the quantity is given, then find how much of the other commodity at the rate proposed may be had for that sum.

2. *If the quantities of both commodities be given, and it should be required to find how much of some other commodity, or how much money should be given for the inequality of their values:* Find the separate value of the two given commodities, subtract the less from the greater, and the remainder will be the balance, or value of the other commodity.

3. *If one commodity is rated above the ready money price, to find the bartering price of the other:* Say, as the ready money price of the one is to the bartering price, so is *that* of the other to *its* bartering price.

EXAMPLES.

1. How much coffee at 25 cents per lb. can I have for 56lb. of tea at 43 cents per lb.?

OPERATION.

```
     5 6 lb. of tea.
     ,4 3 per lb
     ----
     1 6 8
   2 2 4
   -------- lb. oz.
2 5)2 4,0 8(96,5 3/25 Ans.
   2 2 5
   -----
     1 5 8
     1 5 0
     -----
         8
       1 6
     -----
2 5)1 2 8
     1 2 5
     -----
```

2. I have 760 gallons of molasses, at 37 cents 5 mills per gallon, which I would exchange for 66 cwt. 2 qr. of cheese at 4 dollars per cwt. Must I pay or receive money, and how much?

Ans. must receive $19.

3. A and B barter; A has 150 bushels of wheat at 5*s.* 9*d.* per bushel, for which B gives 65 bushels of barley, worth 2*s.* 10*d.* per bushel, and the balance in oats at 2*s.* 1*d.* per bushel; what quantity of oats must A receive from B?

Ans. 325$\frac{15}{25}$ *bushels.*

4. A has linen cloth worth 20*d.* an ell, ready money; but in barter he would have two shillings; B has broad cloth worth 14*s.* 6*d.* per yard, ready money; at what price ought the broad cloth to be rated in barter?

Ans. 17*s.* 4*d.* 3*q.*$\frac{4}{20}$ *per yard.*

SUPPLEMENT TO BARTER.

QUESTIONS.

1. What is Barter?
2. When and how does this rule become useful to merchants?
3. When a given quantity of one commodity is bartered for some other commodity, how is the quantity that will be required of this last commodity found?
4. If the quantity of both commodities be given, and it be required to know how much of some other commodity, or how much money must be given for the inequality, what is the method of procedure?
5. If one commodity be rated above the money price, how do you proceed to find the bartering price of the other commodity?
6. How is Barter proved?

EXERCISES.

1. A and B bartered; A had 41 cwt. of hops, 30*s*. per cwt. for which B gave him £20 in money, and the rest in prunes, at 5*d*. per lb. I demand how many prunes B gave A, besides the £20. *Ans*. 17*C*. 3*qrs*. 4*lb*.

2. How much wine at $1,28 per gallon, must I have for 26cwt. 2qr. 14lb. of raisins, at $9,444 per cwt.? *Ans*. 196*gal*. 1*qt*. 1½*pt*.

§ 7. LOSS AND GAIN.

"Loss and Gain is a rule which enables merchants to estimate the profit or loss in buying and selling goods; also to raise or fall the price of them, so as to gain or lose so much per cent."

CASE I.

To know what is gained or lost per cent. First, find what the gain or loss is by subtraction, then as the price it cost is to the gain or loss, so is \$100 (or £100) to the gain or loss per cent.

EXAMPLES.

1. If I buy candles at 16 cents 7 mills per lb. and sell them at 20 cts. per lb. what shall I gain per cent or in laying out 100 dollars?

OPERATION.

I sell at ,20 per lb.
Bought at ,167 per lb.

I gain ,033 per lb.

```
Then as ,167 : ,0 3 3 : : 100
                 1 0 0
                 ------ D. cts.
        ,167)3,3 0 0(19,76 Ans.
             1 6 7
             -------
             1 6 3 0
             1 5 0 3
             -------
               1 2 7 0
               1 1 6 9
               -------
                 1 0 1 0
                 1 0 0 2
                 -------
                       8
```

2. Bought indigo at \$1,20 per lb. and sold the same at 90 cents per lb. what was lost per cent? *Ans.* \$25.

3. Bought 37 gallons of Brandy at \$1,10 per gallon, and sold it for \$40; what was gained or lost per cent? *Ans.* \$1,719 loss

4. Bought hats at 4*s.* a piece, and sold them again at 4*s.* 9*d.*; what is the profit in laying out £100? *Ans.* £18,15*s.*

CASE II.

To know how a commodity must be sold to gain or lose so much per cent; As 100 dollars (or £100) is to the price; so is 100 dollars (or £100) with the profit added or the loss subtracted to the gaining or losing price.

EXAMPLES.

1\. If I buy wheat at $1,25 per bushel, how must I sell it to gain 15 per cent?

OPERATION.

```
As 100 : 1,2 5 : : 115
         1 1 5
        ------
         6 2 5
       1 2 5
     1 2 5
    ----------- D. cts. m.
 100)1 4 3,7 5(1,43  7 Ans.
     1 0 0
     -----
       4 3 7
       4 0 0
       -----
         3 7 5
         3 0 0
         -----
           7 5 0
           7 0 0
           -----
             5 0
```

2\. If a barrel of rum cost 15 dollars, how must it be sold to lose 10 per cent? *Ans.* $13,50.

3\. If 120lb. of steel cost £7, how must I sell it per lb. to gain 15¼ per cent? *Ans.* 1*s.* 4*d.* per lb.

SUPPLEMENT TO LOSS AND GAIN.

QUESTIONS.

1. What is Loss and Gain?
2. Having the price at which goods are bought and sold, how is the loss or gain estimated?
3. To know how much a commodity must be valued at to gain or lose so much per cent, what is the method of procedure?
4. How may questions in Loss and Gain be proved?

EXERCISES.

1. A draper bought 100 yards of broadcloth for £56. I demand how he must sell it per yard to gain £15 in laying out £100?

Ans. 12*s.* 10*d.* 2*q.*

2. Bought 30 hogsheads of molasses at $600; paid in duties $20,66; for freight $40,78; for porterage $6,05, and for insurance, $30,84; If I sell it at $26 per hogshead, how much shall I gain per cent?

Ans. $11,695.

§ 8. DUODECIMALS;

OR, CROSS MULTIPLICATION.

This rule is particularly useful to Workmen and Artificers in casting up the contents of their work.

Dimensions are taken in feet, inches and parts. Inches and parts are sometimes called primes (′) seconds (″) thirds (‴) and fourths (⁗)

TABLE.

12 *Fourths make* 1 *Third.*
12 *Thirds* - - - 1 *Second.*
12 *Seconds* - - - 1 *Inch or prime.*
12 *Inches or Pr.* 1 *Foot.*

By this rule also may be calculated the solid contents of bodies, having the measures of their different sides, and is very useful therefore in measuring wood.

RULE.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2 Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry an unit for every 12, from each lower denomination to its superior.

3. In the same manner multiply the multiplicand, by the inches in the multiplier, and write the result of each term in the multiplicand, thus multiplied, *one place* to the right hand in the product.

4. Proceed in the same manner with the other parts in the multiplier, which if seconds, write the result *two places* to the right hand; if thirds, *three places*, &c. and their sum will be the answer required.

The more easily to comprehend the rule—NOTE. Feet multiplied by feet give feet—Feet multiplied by inches give inches—Feet multiplied by seconds give seconds—Inches multiplied by inches give seconds—Inches multiplied by seconds give thirds—Seconds multiplied by seconds give fourths.

EXAMPLES.

1. Multiply 7 feet, 3 inches, 2 seconds, by 1 foot, 7 inches and 3 seconds.

OPERATION.

	F.	I.	″		
	7	3	2		
	1	7	3		
	7	3	2 ‴		
	4	2	10	2 ⁗	
		1	9	9	6
Prod.	11	7	9	11	6

Here I multiply the 7f. 3in. 2″ by the 1f. in the multiplier, which gives seconds, inches and feet.

Next I multiply the same 7f. 3in. 2″ by the 7in. saying 7 times 2 is 14 which is once 12 and 2 over, which (2) I set down one place to the right hand that is in the place of thirds and carry one to the next place and proceed in the same manner with the other terms. Lastly, I multiply the multiplicand by the 3″ saying 3 times 2 are 6, which I set down two places to the right hand and so proceed with the other terms of the multiplicand. The sum of all the products is the answer.

2

F. I.		F. I.	
7 5	}	27 9 9	*Prod.*
3 9			

3

F. I.		F. I.	
4 6	}	25 6	*Prod.*
5 8			

4

F. I.		F. I.	
8 3	}	52 3	*Prod.*
6 4			

5. Multiply 7f. 1in. 9″ by 7f 8in. 9″
Prod. 55f. 2in. 9″ 3‴ 9⁗

6. Multiply 9f. 8in. 7″ by 12f. 3in. 10″
Prod. 119f. 8′ 2″ 10‴ 10⁗

7. How much wood in a load which measures 10f. in length, 3f. 9in. in width, and 4f. 8in. in height; and how much will it cost at $1,33 per cord? *Ans. 1 cord and 47 solid feet over—it will cost* $1 81cts. 8m.

Or, we may multiply by the feet as already directed, and for the inches, take such parts of the multiplicand, &c. as the inches are aliquot or even parts of a foot as done in the rule of Practice.

8. How many square feet in a board of 16 feet 4 inches in length, and 2 feet 8 inches wide?

OPERATION.

	Ft.	In.	
6 inches is ½	16	4	
	2	8	
	32	8	
2———⅓	8	2	″
	2	8	8
Ans.	43	6	8

Here in the first place, I multiply the 16ft. 4in. by the feet (2) of the multiplier; the inches (8) not being an even part of a foot, I take such as are an even part; thus, 6in. is half a foot, therefore divide the multiplicand by 2 for 6 inches, and that quotient by 3 (2in. is ⅓ of 6in.) for 2 inches, all which being added give the product of 16 feet 4 inches, multiplied by 2 feet 8 inches.

9. Another board is 18 feet 9 inches in length, and 2 feet 6 inches wide, how many square feet does it contain? *Ans.* 46ft. 10in. 6″

By Practice. *By Duodecimals.*

10. There is a stock of 15 boards, 12 feet 8 inches in length, and 13 inches wide; how many feet of boards does the stock contain?

Ans. 205ft. 10in.

By Practice. *By Duodecimals.*

SUPPLEMENT TO DUODECIMALS.

QUESTIONS.

1. Of what use are Duodecimals ? To whom more specially are they useful ?
2. In what are dimensions taken ?
3. How do you proceed in the multiplication of duodecimals ?
4. For what number do you carry ?
5. What do you observe in regard to setting down the product different from what is common in the multiplication of other numbers ?
6. Of what term is the product which arises from the multiplication of feet by inches ? feet by seconds ? inches by inches ? inches by seconds ? seconds by seconds ?
7. In what way can the operation be varied ?

EXERCISES.

1. Multiply 76 feet 3 inches 9 seconds by 84 feet 7 inches 11 seconds.

OPERATION.

	F.	I.	″		
6 *inches is* ½)	76	3	9		
	84	7	11		
76× 4=	304	0	0		
76× 8=	608	0	0		
3×84=	21	0	0		
9×84=	5	3	0	‴	
I. 1½)	38	1	10	6	
″ 6½)	6	4	3	9	⁗
3½) & 2½)	3	2	1	10	6
	1	7	0	11	3
	1	0	8	7	6
Prod.	6460	7	1	8	3

3. How many square feet in a stock of 12 boards, 17ft. 7′ long, and 1ft. 5in. wide ? *Ans.* 298ft. 11′

2. What is the product of 371ft. 2in. 6 seconds, multiplied by 181ft. 1in. 9″ ?

Ans. 67242ft. 10in. 1″ 4‴ 6⁗

4. How many cubic feet of wood in a load 6ft. 7′ long, 3ft. 5′ high and 3ft. 8′ wide ? *Ans.* 82ft. 5′ 8″ 4‴

The dimensions of wainscotting, paving, plastering and painting are taken in feet and inches, and the contents given in yards.

PAINTERS AND JOINERS

To find the dimensions of their work, take a line and apply one end of it to any corner of the room, then measure the room, going into every corner with the line, till you come to the place where you first began; then see how many feet and inches the string contains; this call the *Compass* or *Round*, which multiplied into the height of the room and the product divided by 9, the quotient will be the contents in yards.

EXAMPLES.

1. If the height of a room painted be 12ft. 4in. and the compass 84ft. 11in. How many square yards does it contain? *Ans.* 116Y. 3ft. 3' 8"

2. There is a room wainscotted, the compass of which is 47ft. 3' and the height 7ft. 6'. What is the content in square yards? *Ans.* 39Y 3ft. 4' 6"

GLAZIER'S WORK BY THE FOOT.

To find the dimensions of their work, multiply the height of windows by their breadth.

EXAMPLE.

There is a house with 4 tiers of windows, and 4 windows in a tier; the height of the first tier is 6ft. 8'; of the second 5ft. 9'; of the third 4ft. 6'; and of the fourth 3ft. 10'; and the breadth of each is 3ft. 5'—What will the glazing come to at 19 cents per foot? *Ans.* $53,88.

§ 9. ALLIGATION.

ALLIGATION is the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality. It is of two kinds, *Medial* and *Alternate*.

ALLIGATION MEDIAL.

Alligation Medial is when the quantities and prices of several things are given to find the mean price of the mixture compounded of those things.

RULE.

As the sum of the quantities or whole composition is to their total value, so is any part of the composition to its value or mean price.

EXAMPLES.

1. A farmer mingled 19 bushels of wheat at 6*s.* per bushel, and 40 bushels of rye at 4*s.* per bushel, and 12 bushels of barley at 3*s.* per bushel together. I demand what a bushel of this mixture is worth ?

OPERATION.

Bush.		*s.*	£.	*s.*
19	*Wheat at,* .	6 *is*	5	14
40	*Rye* —	4 —	8	
12	*Barley* —	3 —	1	16
sum of the — *simples* 71	*Total value.*		15	10

Bush.		£.	*s.*		*Bush.*
As 71	:	15	10	: :	1

$$
\begin{array}{r}
20 \\
\hline
71)310(4s.\ 4d.\ 1\tfrac{41}{71}q.\ \textit{Ans} \\
284 \\
\hline
26 \\
12 \\
\hline
)312(4d. \\
284 \\
\hline
28 \\
4 \\
\hline
)112(1q. \\
71 \\
\hline
41
\end{array}
$$

2. A Refiner having 5*lb.* of silver bullion, of 8*oz.* fine, 10*lb.* of 7*oz.* fine, and 15*lb.* of 6*oz.* fine, would melt all together ; I demand what fineness 1*lb.* of this mass shall be ?

Ans. 6*oz.* 13*pwts.* 8*grs. fine.*

ALLIGATION ALTERNATE,

Is the method of finding what quantity of any number of simples, whose rates are given will compose a mixture of a given rate, it is therefore, the reverse of Alligation Medial, and may be proved by it.

RULE.

1. Write the prices of the simples, the least uppermost, &c. in a column under each other.

2. Connect with a continued line the price of each simple or ingredient, which is less than that of the compound, with one or any number of those that are greater than the compound, and each greater rate or price with one or any number of those that are less.

3. Write the difference between the mean rate or price and that of each of the simples opposite to the rates with which they are connected.

4. Then if only one difference stand against any rate it will be the quantity belonging to that rate, but if there be several, their sum will be the quantity.

NOTE. Questions in this rule admit of as many various answers as there are various ways of connecting the rates of the ingredients together.

EXAMPLES.

A goldsmith would mix gold of 18 carats fine with some of 16, 19, 22, and 24 carats fine, so that the compound may be 20 carats fine; what quantity of each must he take?

OPERATION.			PROOF. oz.	car. *fine.*		
Mix 20 *car.*	16	4	4 of gold at	16	*Ans.*	16×4=64
	18	2	2 - - - - -	18		18×2=36
	19	2	2 - - - - -	19		19×2=38
	22	2×1	3 - - - - -	22		22×3=66
	24	4	4 - - - - -	24		24×4=96
			15 ——	20 *carats fine.*	*oz.*	15)300(20 *car. fine.*

2. A druggist had several sorts of tea, viz. one sort at 12*s.* per lb. another sort at 11*s* a third at 9*s.* and a fourth at 8*s.* per lb. I demand how much of each sort he must mix together, that the whole quantity may be afforded at 10*s.* per lb.

	lb.	*s.p.lb.*		*lb.*	*s.p.lb.*		*lb.*	*s.p.lb.*
1 *Ans.*	2 at	12	2 *Ans.*	3 at	12	3 *Ans.*	1 at	12
	1 at	11		2 at	11		2 at	11
	1 at	9		2 at	9		2 at	9
	2 at	8		3 at	8		1 at	8

	lb.	*s.p.lb.*		*lb.*	*s.p.lb.*		*lb.*	*s.p.lb.*
4 *Ans.*	1 at	12	5 *Ans.*	3 at	12	6 *Ans.*	2 at	12
	3 at	11		1 at	11		3 at	11
	3 at	9		3 at	9		1 at	9
	1 at	8		2 at	8		3 at	8

7. *Ans.* 3*lb. of each sort.*

NOTE. These *seven* answers arise from as many different ways of linking the rates of the simples together.

CASE 2.

When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixture are given to find the several quantities of the rest, in proportion to the given quantity; take the difference between each price and the mean rate as before. Then say,

As the difference of that simple whose quantity is given,
Is to the given quantity,
So is the rest of the differences severally;
To the several quantities required.

EXAMPLES.

1. How much wine, at 80 cents, at 88, and 92 cents per gallon must be mixed with four gallons of wine at 75 cents per gallon, so that the mixture may be worth 86 cents per gallon?

OPERATION.

86	75	$6+2=8$	*stands against the given quantity.*
	80	$2+6=8$	
	88	$6+11=17$	
	92	$11+6=17$	

	gal.		*cts.*	
As 8 : 4 ::	8	:	4 *at* 80	
	17	:	$8\frac{1}{2}$ — 88 *per gal.*	*The answer.*
	17	:	$8\frac{1}{2}$ — 92	

2. A man being determined to mix 10 bushels of wheat at 4*s.* per bushel, with rye at 3*s.* with barley at 2*s.* and with oats at 1*s.* per bushel; I demand how much rye, barley and oats must be mixed with the 10 bushels of wheat that the whole may be sold at 28*d.* per bushel?

	B.p.			*B.*			*B.*	
1 *Ans.*	2 2	of Rye	2 *Ans.*	40	of Rye	3 *Ans.*	8	of Rye
	5 0	- Barley		50	- Barley		10	- Barley
	12 2	- Oats		20	- Oats		14	- Oats
	B.			*B.p.*			*B.*	
4 *Ans.*	10	of Rye	5 *Ans.*	12 2	of Rye	6 *Ans.*	2	of Rye
	10	- Barley		5 0	- Barley		14	- Barley
	15	- Oats		17 2	- Oats		10	- Oats

	B.	
7. *Ans.*	50	of Rye
	70	- Barley
	20	- Oats

Case 3.

When the rates of the several ingredients, the quantity to be compounded, and the mean rate of the whole mixture are given, to find how much of each sort will make up the quantity, find the differences between the mean rate, &c. as in Case 1. Then,

As the sum of the quantities or differences,
Is to the given quantity or whole composition;
So is the difference of each rate,
To the required quantity of each rate.

EXAMPLES.

1. How many gallons of water, of no value, must be mixed with brandy, at one dollar twenty cents per gallon, so as to fill a vessel of 75 gallons, that may be afforded at 92 cents per gallon?

OPERATION.

$$92 \left\{ \begin{array}{rr} \textit{Gal.} & \\ 0 & 28 \\ 1{,}20 & 92 \\ \hline & \textit{Sum } 120 \end{array} \right. \quad \begin{array}{c} \textit{Gal.} \quad \textit{Gal.} \\ \text{As } 120 : 75 :: \end{array} \left\{ \begin{array}{l} \textit{Gal.} \quad \textit{Gal.} \\ 28 : 17\tfrac{1}{2} \textit{ of Water.} \\ 92 : 57\tfrac{1}{2} \textit{ - Brandy.} \\ \hline 75 \textit{ given quantity.} \end{array} \right.$$

2. Suppose I have 4 sorts of currants of 8*d.* 12*d.* 18*d.* and 22*d.* per lb. of which I would mix 120lb. and so much of each sort as to sell them at 16*d.* per lb. how much of each must I take?

$$\textit{Ans.} \left\{ \begin{array}{rcr} \textit{lb.} & \textit{at} & \textit{d} \\ 36 & — & 8 \\ 12 & — & 12 \\ 24 & — & 18 \\ 48 & — & 22 \end{array} \right\} \text{per lb.}$$

3. A Grocer has currants of 4*d.* 6*d.* 9*d.* and 11*d.* per lb. and he would make a mixture of 240lb. so that it might be afforded at 8*d.* per lb. how much of each sort must he take?

$$\textit{Ans.} \left\{ \begin{array}{rcr} \textit{lb.} & \textit{at} & \textit{d.} \\ 72 & — & 4 \\ 24 & — & 6 \\ 48 & — & 9 \\ 96 & — & 11 \end{array} \right\} \text{per lb.}$$

SUPPLEMENT TO ALLIGATION.

QUESTIONS.

1. What is Alligation?
2. Of how many kinds is Alligation?
3. What is Alligation MEDIAL?
4. What is the rule for operating?
5. What is Alligation ALTERNATE?
6. When a number of ingredients of different prices are mixed together, How do we proceed to find the mean price of the compound or mixture?
7. When one of the ingredients is limited to a certain quantity, what is the method of procedure?
8. When the whole composition is limited to a certain quantity, how do you proceed?
9. How is Alligation proved?

EXERCISES.

1. A Grocer would mix three sorts of sugar together; one sort at 10*d*. per lb. another at 7*d*. and another at 6*d*. how much of each sort must he take that the mixture may be sold for 8*d*. per lb.?

Ans. 3lb. at 10*d. 2lb. at 7d. & 2lb. at* 6*d.*

2. A Goldsmith has several sorts of gold; some of 24 carats fine, some of 22 and some of 18 carats fine, and he would have compounded of these sorts the quantity of 60*oz*. of 20 carats fine; I demand how much of each sort must he have?

Ans. 12*oz.* 24 *carats fine,* 12 *at* 22 *carats fine, and* 36 *at* 18 *car. fine.*

§ 10. POSITION.

POSITION is a rule which by false or supposed numbers, taken at pleasure, discovers the true one required. It is of two kinds, *Single* and *Double.*

SINGLE POSITION.

Is the working of one supposed number, as if it were the true one, to find the true number.

RULE.

1 Take any number and perform the same operations with it as are described to be performed in the question.

2. Then say as the sum of the errors is to the given sum, so is the supposed number to the true one required.

Proof. Add the several parts of the sum together, and if it agree with the sum, it is right.

EXAMPLES.

1. Two men, A and B, having found a bag of money, disputed who should have it. A said the half, third, and one fourth of the money made 130 dollars, and if B could tell how much was in it, he should have it all, otherwise he should have nothing; I demand how much was in the bag?

OPERATION.

Suppose 60 *dollars.*

The half	30
— third	20
— fourth	15
	65

As 65 : 130 : : 60
60

65)7800(120 dolls. *the answer.*
65

130
130

000

2 A B and C talking of their ages, B said his age was once and a half the age of A; and C said his age was twice and one tenth the age of both, and that the sum of their ages was 93; what was the age of each?
A's 12, B's 18, C's 63 years.

3. A person having spent ½ and ⅓ of his money, had £26⅔ left, what had he at first? *Ans.* £160.

4. Seven eighths of a certain number exceeds four fifths by 6; what is that number? *Ans.* 80.

DOUBLE POSITION.

DOUBLE POSITION is that which discovers the true number, or number sought, by making use of two supposed numbers.

RULE.

1. Take any two numbers and proceed with them according to the conditions of the question.
2. Place each error against its respective position or supposed number; if the error be too great, mark it with +; if too small, with —
3. Multiply them cross-wise, the first position by the last error, and the last position by the first error.
4. If they be alike, that is, both greater, or both less than the given number, divide the difference of the products by the difference of the errors, and the quotient will be the answer; but if the errors be unlike, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

EXAMPLES.

1. A man lying at the point of death, left to his three sons all his estate, viz. to F half wanting 50 dollars; to G one third; and to H the rest, which was 10 dollars less than the share of G. I demand the sum left, and each son's share.

OPERATION.

Suppose the sum 300 dollars.

Then 300÷2—50=100 F's part.
300÷ 3=100 G's part.
G's part 100—10= 90 H's part.

Sum of all their parts 290

Error 10—

Again suppose the sum 900 dollars.

Then 900÷2—50=400 F's part.
900÷ 3=300 G's part.
G's part 300—10=290 H's part.

Sum of all their parts 990

Error 90+

Suppose.	Errors.
300	10—
X	
900	90+
9000	27000
27000	

——— Dollars.

Sum of Errors. } 100)36000(360 *Ans.*

The divisor is the sum of the errors 90+and 10—

Having proceeded with the supposed numbers according to the conditions of the question, the *sum of all their parts* must be subtracted from the supposed number thus the 290 is subtracted from 300, the supposed number, &c.

2. There is a fish whose head is 10 feet long; his tail as long as his head and half the length of his body, and his body as long as his head and tail; what is the whole length of the fish? *Ans.* 80 feet

3. A certain man having driven his swine to market, viz. hogs, sows and pigs, received for them all £50 being paid for every hog 18*s*. for every sow 16*s*. for every pig 2*s*.; there were as many hogs as sows, and for every sow there were three pigs; I demand how many there were of each sort? *Ans. 25 hogs, 25 sows, and 75 pigs.*

4. A and B laid out equal sums of money in trade; A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost 225 dollars; then A's money was double that of B's; what did each one lay out? *Ans. $600.*

5. A and B have the same income, A saves $\frac{1}{8}$ of his; but B by spending 30 dollars per annum more than A, at the end of 8 years finds himself 40 dollars in debt; what is their income, and what does each spend per annum?

Ans. Their income is $200 per ann. A spends $175, & B 205 per ann.

§ 11. DISCOUNT.

DISCOUNT is an allowance made for the payment of any sum of money before it becomes due, and is the difference between that sum, due sometime hence, and its present worth.

The *present worth* of any sum or debt due some time hence, is such a sum as, if put to interest, would in that time and at the rate per cent for which the discount is to be made, amount to the sum or debt then due.

RULE.

As the amount of 100 dollars for the given time and rate is to 100 dollars, so is the given sum to its present worth, which subtracted from the given sum leaves the discount.

EXAMPLES.

1. What is the discount of \$321,63 due 4 years hence at 6 per cent?

OPERATION.

Dolls.
6 interest of 100 dolls.
4 years. [1 year.

24
100

124 amount.

Then, as 124 : 100 : : 321,63
321,63

.124)32163,00(259,379

321,63 given sum.
259,379 present worth.

Ans. 62,251 discount.

2. What is the present worth of \$426 payable in 4 years and 12 days, discounting at the rate of 5 per cent? *Ans.* \$354,519.

§ 12. EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the finding of a time to pay at once, several debts due at different times, so that neither party shall sustain loss.

RULE.

Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time.

EXAMPLES.

1. A owes B 136 dollars to be paid in 10 months; 96 dollars to be paid in 7 months; and 260 to be paid in 4 months; what is the equated time for the payment of the whole?

OPERATION.

$$
\begin{array}{rr}
136\times & 10=1360 \\
96\times & 7=\ \ 672 \\
260\times & 4=1040 \\
\hline
492 & 3072
\end{array}
$$

$$
\begin{array}{l}
492)3072(6 \text{ months.} \\
\quad 2952 \\
\quad \overline{\ \ \ 120} \\
\quad\ \ \ \ 30 \\
\hline
492)3600(7 \text{ days.} \\
\quad 3444 \\
\quad \overline{\ \ \ 156}
\end{array}
$$

2. I owe \$65,125, to be paid $\frac{1}{4}$ in 3 months, $\frac{1}{2}$ in 5 months, $\frac{1}{8}$ in 10 months, and the remainder in 14 months, at what time ought the whole to be paid?

Ans. $6\frac{1}{4}$ months

3. A merchant has owing to him £300, to be paid as follows; £50 at 2 months, £100 at 5 months, and the rest at 8 months; and it is agreed to make one payment of the whole; I demand when that time must be? *Ans.* 6 months.

4. A merchant owes me 900 dollars, to be paid in 96 days, 130 dollars in 120 days, 500 dollars in 80 days, 1267 dollars in 27 days; what is the mean time for the payment of the whole?

Ans. 63 days, very nearly.

§ 13. GUAGING.

GUAGING is taking the dimensions of a cask in inches to find its contents in gallons by the following

METHOD.

1. Add two thirds of the difference between the head and bung diameters to the head diameter for the mean diameter; but if the staves be but little curving from the head to the bung, add only six tenths of this difference.

2. Square the mean diameter, which multiplied by the length of the cask, and the product divided by 294, for wine, or by 359 for ale, the quotient will be the answer in gallons.

EXAMPLES.

1. How many ale or beer gallons will a cask hold, whose bung diameter is 31 inches, head diameter 25 inches, and whose length is 36 inches?

OPERATION.

```
31 Bung diam.    25 head diam.
25 Head diam.     4 Two thirds difference.
——               ——
 6 Difference.   29 Mean diam.
                 29
                ———
                261
                58
                ———
                841 Square of mean diam.
                 36 Length.
                ———
               5046
              2523
              ————
      359)30276(84 galls. 1 121/359 qts.
```

NOTE 1. In taking the length of the casks, an allowance must be made for the thickness for both heads of one inch, $1\frac{1}{2}$ inches, or 2 inches according to the size of the cask.

NOTE 2. The head diameter must be taken close to the chimes, and for small casks, add 3 tenths of an inch; for casks of 40 or 50 gallons, 4 tenths, and for larger casks, 5 or 6 tenths, and the sum will be very nearly the head diameter within.

§ 14. MECHANICAL POWERS.

1. *OF THE LEVER.*

To find what weight may be raised or balanced by any given power, Say as the distance between the body to be raised or balanced, and the *fulcrum* or *prop*, is to the distance between the prop and the point where the power is applied; so is the power to the weight which it will balance or raise.

EXAMPLE.

IF a man weighing 150lb. rest on the end of a lever 12 feet long, what weight will he balance on the other end, supposing the prop $1\frac{1}{2}$ feet from the weight?

12 *feet the Lever.*
1,5 *distance of the weight from the fulcrum.*

10,5 distance from the fulcrum to the man. Therefore,

	Feet.		*Feet.*		*lb.*		*lb.*	
As	1,5	:	10,5	: :	150	:	1050	*Ans.*

2. *OF THE WHEEL AND AXLE.*

As the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel, to the weight suspended by the axle.

EXAMPLES.

1. A mechanic wishes to make a windlass in such a manner, as that 1lb. applied to the wheel should be equal to 12 suspended on the axle; now supposing the axle 4 inches diameter, required the diameter of the wheel?

	lb.		*in.*		*lb.*		*in.*	
As	1	:	4	: :	12	:	48	*Ans. or diameter of the wheel.*

2. Suppose the diameter of the axle 6 inches, and that of the wheel 60 inches, what power at the wheel will balance 10lb. at the axle? *Ans.* 1lb.

3. *OF THE SCREW.*

The power is to the weight to be raised as the distance between 2 threads of the screw is to the circumference of a circle described by the power applied at the end of the lever.

NOTE 1. *To find the circumference of the circle described by the end of the lever*, multiply the double of the lever by 3,14159, the product will be the circumference.

NOTE 2. It is usual to abate $\frac{1}{3}$ of the effect of the machine for friction.

EXAMPLES.

There is a screw whose threads are an inch asunder; the lever by which it is turned, is 36 inches long, and the weight to be raised a ton, or 2240lb. What power or force must be applied to the end of the lever, sufficient to turn the screw that is to raise the weight?

The lever $36\times2=72\times3,14159=226,194+$ *the circumference.*

	circumf.		in.		lb.		lb.
Then, as	226,194	:	1	: :	2240	:	9,903

PROBLEMS.

1. *The diameter of a circle being given to find the circumference*, multiply the diameter by 3,14159; the product will be the circumference.

2. *To find the area of a circle, the diameter being given*, multiply the square of the diameter by ,785398; the product is the area.

3. *To measure the solidity of any irregular body, whose dimensions cannot be taken*, put the body into some regular vessel and fill it with water, then taking out the body, measure the fall of water in the vessel; if the vessel be square, multiply the side by itself, and the product by the fall of water, which gives the solid contents of the irregular body.

MISCELLANEOUS QUESTIONS.

1. THE Northern Lights were first observed in London 1560; how many years since?

2. What number multiplied by 43 produces 88150? *Ans.* 2050.

3. If a cannon may be discharged twice with 6lb. of powder, how many times will 7*C.* 3*qrs.* 17*lbs.* discharge the same piece? *Ans.* 295 *times.*

4. Reduce 14 guineas and £75 13*s.* 6$\frac{3}{4}$*d.* to Federal Money. *Ans.* \$317,593.

5. What is the interest of \$79,47 one year and five months? *Ans.* \$6,754.

6. A owed B \$317,19, for which he gave his note on interest, bearing date July 12th, 1797. On the back of the note are these several endorsements, viz.

Oct. 17th, 1797, Received in cash \$61,10.

March 20th, 1798, Received 17*Cwt.* beef, at \$4,33 per cwt.

Jan. 1st. 1800, Received in cash, \$84.

What was there due from A to B of principal and interest, Sept. 18th, 801? *Ans.* \$144,363.

7. What cost 13$\frac{1}{2}$ yards of flannel at 1*s.* 8$\frac{1}{2}$*d.* per yard? *Ans.* £1 3*s.* 0$\frac{3}{4}$*d.*

8. What must I give for 3*Cwt.* 2*qrs.* 13*lb.* of cheese at 7 cts. per lb.? *Ans.* \$28,35.

9. What will 35 yards of broadcloth cost at 23*s.* 6*d.* per yard? *Ans.* £41 2*s.* 6*d.*

10. What will be the cost of a loin of veal, weighing 16$\frac{3}{4}$*lb.* at 2$\frac{1}{2}$*d.* per lb.? *Ans.* 3*s.* 5$\frac{3}{4}$*d.*

11. What will 87$\frac{1}{4}$*lb.* of tallow cost at 9$\frac{1}{4}$*d.* per lb.? *Ans.* £3 7*s.* 3*d.*

12. What will 196 yards of tape cost at 3 farthings per yard? *Ans.* 12*s.* 3*d.*

13. What will 56 bushels of oats cost at 2*s.* 3$\frac{1}{2}$*d.* per bushel? *Ans.* £6 8*s.* 4*d.*

14. At £3 7*s.* 6*d.* per cwt. for sugar, what is that per lb.? *Ans.* 7*d.*

15. How much in length of a board that is 10 inches wide will it require to make a square foot? *Ans.* 14$\frac{4}{10}$ *feet.*

16. How many square feet in a board 1 foot 3 inches wide, and 14 feet 9 inches long? *Ans.* 18*f.* 5′ 3″

17. How much wood in a load 9 feet long, 3$\frac{1}{2}$ wide, and 2 feet 9 inches high? *Ans.* 86*f.* 7′ 6″

18. At \$1,33 per yard for cloth, what must I give for 72 yards? *Ans.* \$95,76.

19. If $2\frac{1}{4}$ cwt. of cotton wool cost £11 17s. 6d. what is that per lb ? *Ans.* $11\frac{1}{4}$d.

20. If $1832\frac{1}{4}$ gallons of wine cost £44 6s. what is that per gallon ? *Ans.* $5\frac{3}{4}$d.

21. What will $53\frac{1}{2}$lb. of beef cost at 5cts. 5m. per lb. ? *Ans.* $2,942.

22. What will 50 bushels of potatoes cost at 21 cents per bushel ? *Ans.* $10,50.

23. At $10,76 per cwt. for sugar, what is that per lb. ? *Ans.* $0,096.

24. What will be a man's wages for 6 months, at 43 cents per day, working $5\frac{1}{2}$ days per week ? *Ans.* $61,49.

25. What must I give for pasturing my horse 19 weeks, at 33 cents per week ? *Ans.* $6,27.

26. How many revolutions does the moon perform in 144 years, 2 days, 10 hours. One revolution being in 27 days, 7h. 43m. *Ans.* 1925.

27. What will 7 pieces of cloth containing 27 yards each, come to at 15s. $4\frac{1}{2}$d. per yard ? *Ans.* £145 5s. $10\frac{1}{2}$d.

28. A man spends 23 dolls. 69 cents, 5 mills, in a year, what is that per day ? *Ans.* $0,064.

29. Suppose the Legislature of this State should grant a tax of 7 cents 3 mills on a dollar, what will a man's tax be, who is 142 dollars 40 cents on the list ? *Ans.* $10,395.

30. A Bankrupt, whose effects are 3948 dollars, can pay his creditors but 28 cts. 5 mills on the dollar. What does he owe ? *Ans.* $13852,631.

31. Suppose a cistern having a pipe that conveys 4 gallons, 2qts. into it in an hour, has another that lets out 2 gallons, 1qt. 1pt. in an hour, if the cistern contains 84 gallons, in what time will it be filled ? *Ans.* 39h. 31m. $45\frac{15}{17}$s.

32. If 80 dollars worth of provisions will serve 20 men 25 days, what number of men will the same provisions serve 10 days ? *Ans.* 50 *men.*

33. If 6 men spend 16 dollars 7 cents in 40 days, how long will 135 men be spending 100 dollars ? *Ans.* 11 days, 1h. 30m. 18s.

34. A bridge built across a river in 6 months, by 45 men, was washed away by the current ; required the number of workmen sufficient to build another of twice as much worth in 4 months ? *Ans.* 135 men.

35. Four men, A, B, C, & D, found a purse of money containing 12 dollars, they agree that A shall have one third, B one fourth, C one sixth and D one eighth of it, what must each man have according to this agreement ?
Ans. A's share $$4,571\frac{3}{7}$. B's share, $$3,428\frac{4}{7}$.
C's share $$2,285\frac{5}{7}$. D's share, $$1,714\frac{2}{7}$.

36. A certain usurer lent £90 for 12 months, and received principal and interest £95 8s. I demand at what rate per cent he received interest. *Ans.* 6 *per cent.*

37. If a gentleman have an estate of £1000 per annum, how much may he spend per day to lay up three score guineas at the year's end ? *Ans.* £2 10s. 2d. $1\frac{15}{73}$q.

38. What is the length of a road, which being 33 feet wide contains an acre ? *Ans.* 80 *rods in length.*

39. Required a number from which if 7 be subtracted, and the remainder be divided by 8, and the quotient be multiplied by 5, and 4 added to the product, the square root of the sum extracted, and three fourths of that root cubed, the cube divided by 9, the last quotient may be 24 ? *Ans.* 103.

40. If a quarter of wheat affords 60 ten penny loaves, how many eight penny loaves may be obtained from it ? *Ans.* 75 *eight penny loaves*

41. If the carriage of 7 cwt. 2qr. for 105 miles be £1 5s. how far may 5 cwt. 1qr. be carried for the same money? *Ans.* 150 *miles.*

42. If 50 men consume 15 bushels of grain in 40 days, how much will 30 men consume in sixty days? *Ans.* $13\frac{1}{2}$ *bushels.*

43. On the same supposition, how long will 50 bushels maintain 64 men? *Ans.* 104 *days,* 4 *hours.*

44. A gentleman having 50s. to pay among his laborers for a day's work, would give to every boy 6d. to every woman 8d. and to every man 16d. the number of boys, women and men was the same; I demand the number of each? *Ans.* 20.

45. A gentleman had £7 17s. 6d. to pay among his laborers; to every boy he gave 6d. to every woman 8d. and to every man 16d. and there were for every boy three women, and for every woman 2 men; I demand the number of each? *Ans.* 15 *boys,* 45 *women, and* 90 *men.*

46. Three Gardeners, A, B, and C, having bought a piece of ground, find the profits of it amount to 120£ per annum. Now the sum of money which they laid down was in such proportion, that as often as A paid 5£ B paid 7£ and as often as B paid 4£ C paid 6£. I demand how much each man must have per annum of the gain? *Ans.* A £26 13s. 4d. B £37 6s. 8d. C £56.

47. A young man received 210£ which was 2-3ds. of his eldest brother's portion: now three times the eldest brother's portion was half the father's estate; I demand how much the estate was? *Ans.* £1890.

48. Two men depart both from one place, the one goes North, and the other South; the one goes 7 miles a day, the other 11 miles a day; how far are they distant the 12th day after their departure? *Ans.* 216 *miles.*

49. Two men depart both from one place, and both go the same road the one travels 12 miles every day, the other 17 miles every day; how far are they distant the 10th day after their departure? *Ans.* 50 *miles.*

50. The river Po is 1000 feet broad, and 10 feet deep, and it runs at the rate of 4 miles an hour. In what time will it discharge a cubic mile of water (reckoning 5000 feet to the mile) into the sea? *Ans.* 26 *days,* 1 *hour.*

51. If the country which supplies the river Po with water, be 380 miles long and 120 broad, and the whole land upon the surface of the earth be 62,700,000 square miles, and if the quantity of water discharged by the rivers into the sea be every where proportional to the extent of land by which the rivers are supplied; how many times greater than the Po will the whole amount of the rivers be? *Ans.* 1375 *times.*

52. Upon the same supposition, what quantity of water altogether will be discharged by all the rivers into the sea in a year? *Ans.* 19272 *cubic miles.*

53. If the proportion of the sea on the surface of the earth to that of land be as $10\frac{1}{2}$ to 5, and the mean depth of the sea be a quarter of a mile; how many years would it take if the ocean were empty to fill it by the rivers running at the present rate? *Ans.* 1708 *years,* 17 *days, and* 12 *hours.*

54. If a cubic foot of water weigh 1000 *oz.* avoirdupois, and the weight of mercury be $13\frac{1}{2}$ times greater than of water, and the height of the mercury in the barometer (the weight of which is equal to the weight of a column of air on the same base, extending to the top of the atmosphere) be 30 inches; what will be the weight of the air upon a square foot? a square mile? and what will be the whole weight of the atmosphere, supposing the size of the earth as in questions 51 and 53?

Ans. 2109,375 *pounds weight on a square foot.*
52734375000 - - - - *square miles,*
1024998046875000000 - - - *of the whole atmosphere.*

55. A began trade June 1, with 40 dollars, and took in B as a partner, Sept. 8, following, with 120 dollars; on Dec. 24; A put in 190 dollars more, and continued the whole in trade till May 5, following, when their whole gain was found to be 82 dollars; what is each partner's share?

Ans. A's share $47,065+ *B's share* $34,934+

56. If I give 80 bushels of potatoes at 21 cents per bushel, and 240*lb*. of flax, at 15 cents per lb. for 64 bushels of salt, what is the salt per bushel?

Ans. $0,825.

57 What is the present worth of 482 dollars, payable 4 years hence, discounting at the rate of 6 per cent? *Ans.* $388,709.

58. I have owing to me as follows: viz. $18,73 in 8 months: $46,00 in 5 months; and 104,84 in 3 months; what is the mean time for the payment of the whole? *Ans.* 4 *months* 2 *days.*

59. If I sell 500 deals at 15*d*. a piece, and lose £9 per cent, what do I lose in the whole quantity? *Ans.* £2 16*s*. 3*d*.

60. If I buy 1000 Ells Flemish of linen for £90, what may I sell it per Ell in London, to gain £10 in the whole? *Ans.* 3*s*. 4*d*.

61. How many wine gallons in a cask, whose bung diameter measures 27 inches, head diameter 21 inches, and length 30 inches?

Ans. 63 *gals.* 3*qts.*

62. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and file was just double what it was at first; he was again reinforced with three times his first number of men, and after placing the whole in the same form as at first, his number in rank and file was 40 men each; How many men had he at first? *Ans.* 100.

63. Two ships A and B sailed from a certain port at the same time; A sailed north 8 miles an hour, and B east 6 miles an hour; What was their distance at the end of one hour? *Ans.* 10 *miles.*

64. A hare starts 12 rods before a hound; but is not perceived by him until she has been up 45 seconds; she scuds away at the rate of ten miles an hour, and the dog on view, makes after at the rate of 16 miles an hour. How long will the hound be in overtaking the hare, and what distance will he run? *Ans.* 97½ *seconds, he will run* 2288 *feet.*

65. A fellow said that when he counted his nuts two by two, three by three, four by four, five by five, and six by six, there was still an odd one, but when he counted them seven by seven they came out even; How many had he? *Ans.* 721.

66. There is an island 50 miles in circumference, and three men start together to travel the same way about it; A goes 7 miles per day, B 8, and C 9; when will they all come together again, and how far will they travel? *Ans.* 50 *days.* A 350 *miles.* B 400, *and* C 450.

67. If a weight of 1440lb. be placed 1 foot from the prop, at what distance from the prop must a power of 160lb. be applied to balance it?

Ans. 9 *feet.*

68. Sound, uninterrupted, moves about 1142 feet in a second; supposing in a thunder storm, the space between the lightning and thunder be six seconds; at what distance was the explosion?

Ans. 1 *mile,* 94 *rods,* 21 *feet.*

69. A cannon ball at the first discharge, flies about a mile in *eight* seconds: at this rate, how long would a ball be in passing from the Earth to the Sun, it being, as astronomers well know, 95173000 miles distant?

Ans. 24 *years*, 46 *days*, 7 *hours*, 33*m*. 20*s*.

70. A general disposing his army into a square battalion, found he had 231 over and above; but increasing each side with one soldier, he wanted 44 to fill up the square; Of how many men did his army consist?

Ans. 19000.

71. A and B cleared by an adventure at sea 45 guineas, which was £35 *per cent* upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; what money did each adventure?

Ans. A £104 4*s*. $2\frac{10}{19}$*d*. B £75 15*s*. $9\frac{9}{19}$*d*.

72. Tubes may be made of gold weighing not more than at the rate of $\frac{1}{1652}$ of a grain per foot; what would be the weight of such a tube, which would extend across the Atlantic from Boston to London, estimating the distance at 3000 miles?

Ans. 2*oz*. 6*pwt*. $3\frac{11}{163}$*gr*

PLEASING AND DIVERTING QUESTIONS.

1. There was a well 30 feet deep; a frog at the bottom could jump up 3 feet every day, but he would fall back two feet every night. How many days did it take the frog to jump out?

2. Two men were driving sheep to market, says one to the other, give me one of yours and I shall have as many as you; the other says, give me one of yours and I shall have as many again as you. How many had each?

3. As I was going to St. Ives,
I met seven wives,
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits,
Kits, cats, sacks and wives,
How many were going to St. Ives?

4. The account of a certain school is as follows, viz. 1-16 of the boys learn geometry, 3-8 learn grammar, 3-10 learn arithmetic, 3-20 to write, and 9 learn to read: I demand the number of each?

5. A man driving his geese to market, was met by another, who said, Good morrow master, with your hundred geese; says he, I have not an hundred, but if I had half as many as I now have, and two geese and a half beside the number I now have already, I should have an hundred. How many had he?

6. Three travellers met at a Caravansary, or inn in Persia; and two of them brought their provision along with them, according to the custom of the country; but the third not having provided any, proposed to the others that they should eat together, and he would pay the value of his proportion. This being agreed to, A produces 5 loaves, and B 3 loaves, which the travellers eat together, and C paid 8 pieces of money as the value of his share, with which the others were satisfied, but quarrelled about the dividing of it. Upon this, the affair was referred to the judge, who decided the dispute by an impartial sentence. Required his decision?

7. Suppose the 9 digits to be placed in a quadrangular form; I demand in what order they must stand, that any three figures in a right line may make just 15?

8. A countryman having a Fox, a Goose, and a peck of corn, in his journey, came to a river, where it so happened that he could carry but one over at a time. Now as no two were to be left together that might destroy each other; so he was at his wits end how to dispose of them; for says he, tho' the corn can't eat the goose, nor the goose eat the fox; yet the fox can eat the goose, and the goose eat the corn. The question is, how he must carry them over that they may not devour each other?

9. Three jealous husbands with their wives, being ready to pass by night over a river, do find at the water side a boat which can carry but two persons at once, and for want of a waterman they are necessitated to row themselves over the river at several times: The question is, how those six persons shall pass by 2 and 2, so that none of the three wives may be found in the company of one or two men, unless her husband be present?

10. Two merry companions are to have equal shares of 8 gallons of wine, which are in a vessel containing exactly 8 gallons; now to divide it equally between them, they have only two other empty vessels, of which one contains 5 gallons, and the other 3: The question is, how they shall divide the said wine between them by the help of these three vessels, so that they may have four gallons apiece?

SECTION IV.

FORMS OF NOTES, DEEDS, BONDS, AND OTHER INSTRUMENTS OF WRITING.

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§ 1. OF NOTES.

No. I.

Overdean, Sept. 17, 1802.—For value received I promise to pay to *Oliver Bountiful*, or order, sixty three dollars fifty four cents, on demand, with interest after three months. WILLIAM TRUSTY.

Attest, Timothy Testimony.

No. II.

Bilfort, Sept. 17, 1802.—For value received, I promise to pay to O. R or bearer ——— dollars ——— cents, three months after date.

PETER PENCIL.

No. III.

By two Persons.

Arian, Sept. 17, 1802.—For value received, we jointly and severally promise to pay to C. D. or order, ——— dollars, ——— cents, on demand with interest.

Attest, *Constance Adley.*

ALDEN FAITHFUL.
JAMES FAIRFACE.

OBSERVATIONS.

1. No note is negotiable unless the words, *or order*, otherwise *or bearer*, be inserted in it.

2. If the note be written to pay him "*or order*," (No. 1.) then *Oliver Bountiful* may endorse this note, that is, write his name on the backside and sell it to A, B, C, or whom he pleases. Then A, who buys the note, calls on William Trusty for payment, and if he neglects, or is unable to pay, A may recover it of the endorser.

3. If a note be written to pay him "*or bearer*," (No. 2.) then any person who holds the note may sue and recover the same of Peter Pencil.

4. The rate of interest established by law, being *six per cent per annum*, it becomes unnecessary in writing notes to mention the rate of interest ; it is sufficient to write them for the payment of such a sum, with interest, for it will be understood legal interest, which is six per cent.

5. All notes are either payable on demand or at the expiration of a certain term of time agreed upon by the parties and mentioned in the note as three months, a year, &c.

6. If a bond or note mention no time of payment, it is always on demand, whether the words, *on demand*, be expressed or not.

7. All notes payable at a certain time are on interest as soon as they become due, though in such notes there be no mention made of interest.

This rule is founded on the principle that every man ought to receive his money when due, and that the non payment of it at that time is an injury to him. The law, therefore, to do him justice, allows him interest from the time the money becomes due, as a compensation for the injury.

8. Upon the same principle a note payable on demand, without any mention made of interest, is on interest after a demand of payment, for upon demand such notes immediately become due.

9. If a note be given for a specific article, as rye, payable in one, two, or three months, or in any certain time, and the signer of such note suffers the time to elapse without delivering such article, the holder of the note will not be obliged to take the article afterwards, but may demand and recover the value of it in money.

§ 2. OF BONDS.

A BOND WITH A CONDITION FROM ONE TO ANOTHER.

KNOW all men by these presents, that I, C. D. of &c. in the county of &c. am held and firmly bound to E. F. of &c. in two hundred dollars, to be paid to the said E. F. or his certain attorney, his executors, administrators or assigns, to which payment, well and truly to be made, I bind myself, my heirs, executors and administrators, firmly by these presents; Sealed with my seal. Dated the eleventh day of——in the year of our Lord one thousand eight hundred and two.

The Condition of this obligation is such, that if the above bound C. D. his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid unto the above named E. F. his executors, administrators or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the eleventh day of——next ensuing the date hereof: Then this obligation to be void, or otherwise to remain in full force and virtue.

Signed, &c.

A Condition of a Counter Bond, or Bond of Indemnity, where one man becomes bound for another.

THE condition of this obligation is such, that whereas the above named A. B. at the special instance and request, and for the only proper debt of the above bound C. D. together with the said C. D. is, and by one bond or obligation bearing equal date with the obligation above written, held and firmly bound unto E. F. of &c. in the penal sum of————dollars, conditioned for the payment of the sum of, &c. with legal interest for the same, on the——day of——next ensuing the date of the said in part recited obligation, as in and by the said in part recited bond, with the condition thereunder written may more fully appear. If therefore the said C. D. his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid unto the said E. F. his executors, administrators, or assigns, the said sum of, &c. with legal interest of the same, on the said——day of, &c. next ensuing the date of the said in part recited obligation, according to the true intent and meaning, and in full discharge and satisfaction of the said in part recited bond or obligation: Then, &c, Otherwise, &c.

NOTE. The principal difference between a note and a bond, is that the latter is an instrument of more solemnity, being given under seal. Also, a note may be controuled by a special agreement, different from the note, whereas in case of a bond, no special agreement can in the least controul what appears to have been the intention of the parties as expressed by the words in the condition of the bond.

§ 3. OF RECEIPTS.

No. I.

Sitgrieves, Sept. 19, 1802. Received from Mr. *Durance Adley*, ten dollars in full of all accounts.

ORVAND CONSTANCE.

No. II.

Sitgrieves, Sept. 19, 1802. Received of Mr *Orvand Constance*, five dollars in full of all accounts.

DURANCE ADLEY.

No. III.

Receipt for an endorsement on a Note.

Sitgrieves, Sept. 19, 1802. Received of Mr. Simpson Eastly, (by the hand of Titus Trusty) sixteen dollars twenty five cents, which is endorsed on his note of June 3, 1802.

PETER CHEERFUL.

No. IV.

A Receipt for money received on account.

Sitgrieves, Sept. 19, 1802. Received of Mr. Orand Landike, fifty dol ars on account.

ELDRO SLACKLEY.

No. V.

Receipt for interest due on a Bond.

Received this———day of———of Mr. A. B. the sum of five pounds in full of one year's interest of £100 due to me on the———day of———last on bond from the said A. B. I say received. By me C. D.

OBSERVATIONS.

1. There is a distinction between receipts given in full of *all accounts*, and others in full of *all demands*. The former cut off accounts *only*; the latter cut off not only all accounts, but all obligations and right of action.

2. When any two persons make a settlement and pass receipts (No. I. and II.) each receipt must specify a particular sum received, less or more It is not necessary that the sum specified in the receipt, be the exact sum received.

§ 4. OF ORDERS.

No. I.

Mr. Stephen Burgess,

Sir,

For value received, pay to A. B. Ten Dollars, and place the same to my account. SAMUEL SKINNER.

Arckdale, Sept. 9, 1802.

No. II.

Sir, *Boston, Sept.* 9, 1802.

For value received, pay G. R. eighty six cents, and this with his receipt shall be your discharge from me.

NICHOLAS REUBENS.

To Mr. James Robottom.

§ 5. OF DEEDS.

No. I.

A Warrantee Deed.

KNOW ALL MEN BY THESE PRESENTS, That I, Peter Careful, of Leominster, in the County of Worcester, and Commonwealth of Massachusetts, gentleman, for and in consideration of one hundred and fifty dollars, and forty five cents, paid to me by Samuel Pendleton of Ashby, in the County of Middlesex, and Commonwealth of Massachusetts, yeoman, the receipt whereof I do hereby acknowledge, do hereby give, grant, sell and convey to the said Samuel Pendleton, his heirs and assigns, a certain tract and parcel of land, bounded as follows, viz.

[*Here insert the bounds, together with all the privileges and appurtenances thereunto belonging.*]

To have and to hold the same unto the said Samuel Pendleton, his heirs and assigns to his and their use and behoof forever. And I do covenant with the said Samuel Pendleton, his heirs and assigns, that I am lawfully seized in see of the premises, that they are free of all incumbrances, and that I will warrant and will defend the same to the said Samuel Pendleton, his heirs and assigns forever, against the lawful claims and demands of all persons.

In witness whereof, I hereunto set my hand and seal this —— day of —— in the year of our Lord one thousand eight hundred and two.

Signed, sealed and delivered in presence of PETER CAREFUL. O.
L. R. F. G.

No. II.

Quitclaim Deed.

KNOW ALL MEN BY THESE PRESENTS, That I, A. B. of, &c. in consideration of the sum of —— to be paid by C. D. of &c. the receipt whereof I do hereby acknowledge, have remissed, released, and forever quitclaimed, and do by these presents remit, release, and forever quitclaim unto the said C. D. his heirs and assigns forever (*Here insert the premises.*) To have and to hold the same, together with all the privileges and appurtenances thereunto belonging, to him the said C. D. his heirs and assigns forever. —*In witness, &c.*

No. III. A Mortgage Deed.

KNOW ALL MEN BY THESE PRESENTS, That I Simpson Easley, of—— in the County of——in the State of——Blacksmith, in consideration of——Dollars——Cents, paid by Elvin Fairface of——in the county of —— in the State of —— Shoemaker, the receipt whereof I do hereby acknowledge, do hereby give, grant, sell and convey unto the said Elvin Fairface, his heirs and assigns, a certain tract and parcel of land, bounded as follows, viz. (*Here insert the bounds, together with all the privileges and appurtenances thereunto belonging.*) To have and to hold the afore granted premises to the said Elvin Fairface, his heirs and assigns, to his and their use and behoof forever. And I do covenant with the said Elvin Fairface, his heirs and assigns, That I am lawfully seized in fee of the afore granted premises. That they are free of all incumbrances: That I have good right to sell and convey the same to the said Elvin Fairface. And that I will warrant and defend the same premises to the said Elvin, his heirs and assigns forever, against the lawful claims and demands of all persons. *Provided nevertheless*, That if I the said Simpson Easley, my heirs, Executors, or administrators shall well and truly pay to the said Elvin Fairface, his heirs, executors, administrators or assigns, the full and just sum of——dollars——cents on or before the——day of——which will be in the year of our Lord eighteen hundred and—— with lawful interest for the same until paid, then this deed, as also a certain bond [*or note, as the case may be*] bearing even date with these presents given by me to the said Fairface, conditioned to pay the same sum and interest at the time aforesaid, shall be void, otherwise to remain in full force and virtue. In witness whereof, I the said Simpson and Abigail my wife, in testimony that she relinquishes all her right to dower or alimony in and to the above described premises, hereunto set our hands and seals this ——day of—— in the year of our Lord one thousand eight hundred and five.

Signed, sealed and delivered in presence of L. N. V. X.

SIMPSON EASLEY. O
ABIGAIL EASLEY. O

§ 6. OF AN INDENTURE.

A common Indenture to bind an Apprentice.

THIS Indenture witnesseth, That A. B. of, &c. hath put and placed, and by these presents doth put and bind out his son C. D. and the said C. D. doth hereby put, place and bind out himself, as an apprentice to R. P. to learn the art, trade, or mystery of——The said C. D. after the manner of an apprentice, to dwell with and serve the said R. P.——from the day of the date hereof, until the——day of——which will be in the year of our Lord one thousand eight hundred and——at which time the said apprentice, if he should be living, will be twenty one years of age: During which time or term the said apprentice his said master well and faithfully shall serve; his secrets keep, and his lawful commands every where, and at all times readily obey. He shall do no damage to his said master, nor wilfully suffer any to be done by others; and if any to his knowledge be intended, he shall give his master seasonable notice thereof. He shall not waste the goods of his said master nor lend them unlawfully to any; at cards, dice, or any unlawful game he shall not play; fornication he shall not commit, nor matrimony contract during the said term; taverns, ale houses, or places of gaming he shall not haunt or frequent. From the ser-

vice of his said master he shall not absent himself; but in all things, and at all times he shall carry and behave himself as a good and faithful apprentice ought, during the whole time or term aforesaid.

And the said R. P. on his part doth hereby promise, covenant and agree to teach and instruct the said apprentice, or cause him to be taught and instructed in the art, trade or calling of a——by the best way or means he can, and also teach and instruct the said apprentice, or cause him to be taught and instructed to read and write, and cypher as far as the rule of Three, if the said apprentice be capable to learn, and shall well and faithfully find and provide for the said apprentice, good and sufficient meat, drink, cloathing, lodging and other necessaries fit and convenient for such an apprentice, during the term aforesaid, and at the expiration thereof, shall give unto the said apprentice, two suits of wearing apparel, one suitable for the Lord's day, and the other for working days.

In testimony whereof, the said parties have hereunto interchangeably set their hands and seals, this said——day of——in the year of our Lord one thousand eight hundred and—— (Seal)

Signed, sealed and delivered in presence of (Seal) (Seal)

§ 7. OF A WILL.

The form of a Will with a Devise, of a Real Estate, Leasehold, &c.

In the name of God, Amen, I, A. B. of, &c. being weak in body, but of sound and perfect mind, and memory, (*or you may say thus*, considering the uncertainty of this mortal life, and being of sound, &c.) blessed be Almighty God for the same, do make and publish this as my last Will and Testament in a manner and form following (that is to say) *First*, I give and bequeath unto my beloved wife, J. B. the sum of——I do also give and bequeath unto my eldest son G. B. the sum of——I do also give and bequeath unto my two younger sons J. B. and F. B. the sum of——apiece. I also give and bequeath to my daughter in law, S. H. H. single woman, the sum of——which said several legacies or sums of money, I will and order shall be paid to the said respective legatees within six months after my decease. I further give and devise to my said eldest son G. B. his heirs and assigns, *All* that my messuage or tenement, situate, lying and being in &c. together with all my other freehold estate whatsoever, to hold to him the said G. B. his heirs and assigns forever. And I hereby give and bequeath to my said younger sons J. B. and F. B. all my leasehold estate of and in all those messuages or tenements, with the appurtenances, situate &c. equally to be divided between them. And lastly, as to all the rest, residue and remainder of my personal estate, goods and chattels, of what kind and nature soever, I give and bequeath the same to my said beloved wife J. B. whom I hereby appoint sole executrix of this my last Will and Testament; and hereby revoking all former Wills by me made.

In witness whereof, I hereunto set my hand and seal, this——day of —— in the year of our Lord——

Signed, sealed, published and declared by the above named A. B. to be his last Will and Testament in the presence of us, who have hereunto subscribed our names as witnesses, in the presence of the testator, A. B. (Seal)

R. S.
W. T.
T. W.

APPENDIX.

VULGAR FRACTIONS.

VULGAR FRACTIONS are parts of an unit, or integer; and are represented by two numbers, placed one above the other, with a line drawn between them.

The number above the line is called the *numerator*, and that below the line the *denominator*.

The denominator shews how many parts an integer is divided into, and the numerator shews how many of those parts are meant by the fraction

Fractions are either proper, improper, compound, or mixed.

A *proper fraction* is when the numerator is less than the denominator; as $\frac{1}{3}$, $\frac{3}{7}$, $\frac{17}{28}$, &c.

An *improper fraction* is when the numerator is greater than the denominator; as $\frac{5}{3}$, $\frac{8}{6}$, $\frac{45}{20}$, &c.

A *compound fraction* is the fraction of a fraction, coupled by the word *of*; as $\frac{3}{4}$ of $\frac{2}{5}$, &c.

A *mixed number or fraction* is composed of a whole number and a fraction; as $7\frac{2}{5}$, $28\frac{4}{6}$, &c.

Reduction of Vulgar Fractions

I. To reduce a given fraction to its lowest terms

RULE.

Divide both the numerator and the denominator by some one number that will divide them both without a remainder: divide the quotients in the same manner, and so on till no number greater than 1 will divide them both, and the last quotients express the fraction in its lowest terms.

1. Reduce $\frac{288}{480}$ to its lowest terms.

Thus, 8 $\frac{288}{480} \overset{4)}{=} \frac{36}{60} \overset{3)}{=} \frac{9}{15} = \frac{3}{5}$ *the Answer.*

2. Reduce $\frac{144}{216}$ to its lowest terms. *Ans.* $\frac{2}{3}$.

3. Reduce $\frac{57}{456}$ to its lowest terms. *Ans.* $\frac{1}{8}$.

4. Reduce $\frac{3010}{624}$ to its lowest terms. *Ans.* $\frac{11}{16}$

Or ;—*Find a common measure*, thus,

Divide the denominator by the numerator, and that divisor by the remainder, continuing so to do till nothing remains ; the last divisor is the common measure ; then divide both terms of the fraction by the common measure, * and the quotients will express the fraction required.

1 Reduce $\frac{1080}{1224}$ to its lowest terms.

```
        1080)1224(1
             1080
             ----
              144)1080(7
                  1008
                  ----
Common Measure.     72)144(2
                       144
```

Then $72)\frac{1080}{1224}(\frac{15}{17}$ *the Answer.*

2. Reduce $\frac{308}{4256}$ to its lowest terms. *Ans.* $\frac{11}{152}$.

II. To reduce a mixed number to an improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and to the product add the numerator for a new numerator, and place it over the denominator.

1. Reduce $127\frac{4}{17}$ to an improper fraction.

```
             127
              17
             ---
             889
            127
Numerator,     4
             ---
           2163/17 Ans.
```

Here we multiply the whole number, 127 by 17 the denominator of the fraction, adding in the numerator 4 the sum 2163 is the numerator to the fraction sought, and 17 the denominator, so that $\frac{2163}{17}$ is the improper fraction, equal to $127\frac{4}{17}$.

Reduce $653\frac{3}{19}$ to an improper fraction. *Ans.* $\frac{12410}{19}$.

III. To reduce an improper fraction to its proper terms, or mixed number.

RULE.

Divide the numerator by the denominator, the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

EXAMPLES.

1. Reduce $\frac{15}{6}$ to a mixed number.

$6)15(2\frac{3}{6}$ *Ans.*

2. Reduce $\frac{245}{9}$ to its proper terms. *Ans.* $27\frac{2}{9}$.
3. Reduce $\frac{2163}{17}$ to a mixed number. *Ans.* $127\frac{4}{17}$.
4. Reduce $\frac{12410}{19}$ to a mixed number. *Ans.* $653\frac{3}{19}$.

---

* If the common measure happens to be 1, the fraction is already in its lowest terms.

IV. To find the least common multiple of two or more numbers.

RULE

Divide by any number that will divide two, or more, of the numbers without a remainder, and set the quotients, together wit undivided numbers, in a line underneath.

Divide these quotients and undivided numbers as before, and s till there are no two numbers that can be divided; then the cont product of the divisors and the last quotients together will be the common multiple required.

EXAMPLES.

1. What is the least common multiple of 9, 8, 15 and 16?

8)	9	8	15	16
3)	9	1	15	2
	3	1	5	2

Then $8\times3\times3\times5\times2=720$ *Ans.*

2. What is the least number that 3, 5, 8, and 10 will measure?

Ans. 1

3. What is the least number that can be divided by the nine without a remainder? *Ans.* 25

V. To reduce a compound fraction to a single one.

RULE.

Multiply all the numerators together for a new numerator, and a denominators for a new denominator, then reduce the new fraction lowest term by Case I.

EXAMPLES.

1. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{10}$ to a single fraction.

$$\frac{3\times5\times9}{4\times6\times10}=\frac{135}{240}=\frac{9}{16}\ \textit{Answer.}$$

2. Reduce $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{15}{16}$ to a single fraction. *Ans.* [illegible]

3. Reduce $\frac{2}{15}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20* to a simple (or improper) fracti

Ans. [illegible]

VI. To reduce fractions of different denominations to equivalent frac having a common denominator.

RULE.

Multiply each numerator into all the denominators, except its ow a new numerator, and all the denominators into each other, conti for a common denominator.

---

* Any whole number may be reduced to an improper fraction, by placi der it for a denominator, which must be done in this case, thus $\frac{20}{1}$.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$1\times5\times8=40$ the new numerator for $\frac{1}{4}$
$2\times4\times8=64$ for $\frac{2}{5}$
$5\times4\times5=100$ for $\frac{5}{8}$
$4\times5\times8=160$ the common denominator.

Hence the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$ and $\frac{100}{160}$ the answer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$, and $\frac{3}{13}$ to a common denominator.
Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$.

3. Reduce $\frac{11}{15}$, $\frac{3}{4}$ of $2\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$ to a common denominator.
Ans. $\frac{8448}{11520}$, $\frac{21600}{11520}$, $\frac{6720}{11520}$, and $\frac{7200}{11520}$.

VII. To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULE.

Reduce the given fraction to a compound one by comparing it with all the denominations between it and that denomination you would reduce it to; then reduce that compound fraction to a simple one, by Case V.

EXAMPLES.

1. Reduce $\frac{7}{8}$ of a penny to the fraction of a pound.

By comparing it, the compound fraction will be $\frac{7}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$. Then

$$\frac{7\times1\times1}{8\times12\times20}=\frac{7}{1920}$$ *of a pound, the Answer.*

2. Reduce $\frac{8}{9}$ of a pound Avoirdupois to the fraction of 1 cwt.
Ans. $\frac{1}{126}$ *Cwt.*

3. Reduce $\frac{4}{5}$ of a Pennyweight to the fraction of a pound Troy.
Ans. $\frac{1}{300}$ *lb.*

4. Reduce $\frac{5}{8}$ of a penny to the fraction of a Guinea.
Ans. $\frac{5}{2016}$ *Guinea.*

VIII. To reduce the fraction of one denomination to the fraction of another but less, retaining the same value.

RULE.

Reduce the given fraction to a compound one, as in the preceding case, only observing to invert the parts contained in the integer; then reduce this compound fraction to a simple one, and that to its lowest terms.

EXAMPLES.

1. Reduce $\frac{7}{1920}$ of a pound to the fraction of a penny.

Thus $\frac{7}{1920}$ of $\frac{20}{1}$ of $\frac{12}{1}$. Then $\frac{7\times20\times12}{1920\times1\times1}=\frac{1680}{1920}=\frac{7}{8}$ *d.*

2. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing. *Ans.* $\frac{3}{4}$*q.*
3. Reduce $\frac{7}{1920}$ of a ℔ Troy to the fraction of a *pwt.* *Ans.* $\frac{7}{8}$ *pwt.*
4. Reduce $\frac{4}{7}$ * of a guinea to the fraction of a pound. *Ans.* $\frac{3}{5}$£.

* *Compared thus* $\frac{4}{7}$ *of* $\frac{21}{1}$ *of* $\frac{1}{20}$.

IX. To find the value of a fraction in the known parts of the integer.

RULE.

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination and divide as before, and so on, as far as necessary; the several quotients will be the answer.

EXAMPLES.

1. What is the value of $\frac{2}{3}$ of a pound?

```
   2
  20
3)40
  13s.—1
      12
    3)12
      4d.
```

Ans. 13*s.* 4*d.*

Multiply the numerator of the fraction (2) by 20, the number of shillings in a pound; the product, (40) divided by 3, the denominator, gives 13, the number of shillings, and 1 remaining, being multiplied by 12 the number of pence in a shilling, and the product, (12) divided as before by 3, gives 4, the number of pence, and no remainder. Hence the answer, 13*s.* 4*d.*

2. What is the value of $\frac{3}{5}$ of a pound Troy? *Ans.* 7 *oz.* 4 *pwt.*

3. What is the value of $\frac{4}{5}$ of a pound Avoirdupois? *Ans.* 12 *oz.* $12\frac{4}{5}$ *dr.*

4. Reduce $\frac{4}{5}$ of a mile to its proper quantity. *Ans.* 6 *fur.* 16 *poles.*

X. To reduce any given quantity to the fraction of a greater denomination of the same kind.

RULE.

Reduce the given quantity to the lowest denomination mentioned for a numerator; then reduce the integral part to the same denomination for a denominator, which placed under the numerator before found will express the fraction required.

EXAMPLES.

1. Reduce 16*s.* 8*d.* to the fraction of a pound.

```
 1£. Integral part.          16s. 8d.
20                           12
---                          ---
20                          200 Numerator.
12
---
240 Denominator.
```

Ans. $\frac{200}{240}=\frac{5}{6}$£.

2. Reduce 6 furlongs and 16 poles to the fraction of a mile. *Ans.* $\frac{4}{5}$ *of a mile.*

3. Reduce 12 *oz.* $12\frac{4}{5}$ *dr.* to the fraction of a pound Avoirdupois. *Ans.* $\frac{4}{5}$ *of a pound.*

Addition of Vulgar Fractions.

RULE.

Reduce compound fractions to single ones, mixed numbers to improper fractions, and fractions of different integers to those of the same, and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fraction required.

EXAMPLES.

1. Add $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together.

First, the mixed number $9\frac{1}{5}=\frac{46}{5}$; the compound fraction $\frac{2}{3}$ of $\frac{1}{2}=\frac{2}{6}$ Then the fractions are $\frac{3}{7}$, $\frac{46}{5}$ and $\frac{2}{6}$; which reduce to a common denominator.

$$\begin{aligned} 3\times5\times6 &= 90 \\ 46\times7\times6 &= 1932 \\ 2\times7\times5 &= 70 \\ 7\times5\times6 &= \frac{2092}{210}=9\frac{101}{105} \textit{ Answer.} \end{aligned}$$

2. Add $1\frac{1}{12}$, $2\frac{5}{12}$, $3\frac{7}{12}$ and $14\frac{11}{12}$ together. *Ans.* 22.

3. Add [illegible]£. [illegible]s. and [illegible]d. together. *Ans.* 2s. $8\frac{84}{105}$d.

4. Add $\frac{3}{8}$ of 17£. $9\frac{3}{8}$£. and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{4}{7}$£. together.
Ans. 16£. 12s. $3\frac{3}{7}$d.

Subtraction of Vulgar Fractions.

RULE.

Prepare the fractions as in addition, and the difference of the numerators written above the common denominator will give the difference of the fractions required.

To subtract a fraction from a whole number; from the denominator of the fraction subtract the numerator and place the remainder over the denominator: then deduct 1 from the integer or whole number.

EXAMPLES.

1. From $\frac{49}{50}$ take $\frac{5}{9}$.

$$\begin{aligned} 49\times9 &= 441 \\ 5\times50 &= 250 \\ 50\times9 &= 450 \textit{ com. denom.} \end{aligned}$$

Therefore $\frac{441}{450}-\frac{250}{450}=\frac{191}{450}$ *the Answer*

2. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Ans.* $2\frac{1}{12}$.

3. From $\frac{3}{4}$£. take $\frac{3}{4}$ of a shilling. *Ans.* 14s. 3d.

4 From 7 weeks take $9\frac{7}{10}$ days. *Ans.* 5w. 4d. 7h. 12m.

5 From 3 take $\frac{3}{16}$. *Ans.* $2\frac{13}{16}$.

Multiplication of Vulgar Fractions.

RULE.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then multiply the numerators together for a new numerator, and the denominators for a new denominator.

EXAMPLES.

1. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$.

$4\frac{1}{2}=\frac{9}{2}$. Then $\frac{9\times1}{2\times8}=\frac{9}{16}$ *the Answer.*

2. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. *Ans.* $\frac{5}{14}$.
3. Multiply $48\frac{3}{5}$ by $13\frac{5}{6}$. *Ans.* $672\frac{3}{10}$.
4. Multiply $\frac{9}{10}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{8}$. *Ans.* $\frac{9}{40}$

Division of Vulgar Fractions.

RULE.

Prepare the fractions, as already directed; then invert the divisor and proceed as in multiplication.

EXAMPLES.

1. Divide $\frac{4}{7}$ by $\frac{2}{3}$.

The divisor $\frac{2}{3}$ inverted will be $\frac{3}{2}$. Then $\frac{4\times3}{7\times2}=\frac{12}{14}=\frac{6}{7}$ *Ans.*

2 Divide 5 by $\frac{7}{10}$. *Ans.* $7\frac{1}{7}$.
3. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7. *Ans.* $2\frac{13}{21}$.
4. Divide $\frac{3}{7}$ by 9. *Ans.* $\frac{1}{21}$.
5. Divide 7 by $\frac{3}{8}$. *Ans.* $18\frac{2}{3}$.
6 Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91. *Ans.* $71\frac{1}{2}$.

Rule of Three Direct in Vulgar Fractions

RULE.

Having stated the question, make the necessary preparations, as in Reduction of Fractions, and invert the first term; then proceed as in Multiplication of Fractions.

EXAMPLES

1. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a pound what will $\frac{3}{8}$ of an ell English come to, at the same rate?

First, reduce the $\frac{1}{4}$ of a yard to the fraction of an Ell English; thus $\frac{1}{4}$ of $\frac{4}{1}$ of $\frac{1}{5} = \frac{4}{20}$ E. Eng.

$$\text{Then, as } \overset{\text{E. E.}}{\tfrac{4}{20}} : \overset{\pounds}{\tfrac{2}{3}} :: \overset{\text{E. E.}}{\tfrac{3}{5}}.$$ Invert the first term, and proceed as in Multiplication—

$$\text{Thus } \frac{20 \times 2 \times 3 = 120 =}{4 \times 3 \times 5 = 60 =} \pounds 2.\ \ 0\ \ 0.$$

2. If $\frac{3}{5}$ *yd.* cost $\frac{7}{8}$£. what will $40\frac{1}{2}$ *yds.* come to? *Ans.* £59 1*s.* 3*d.*

3. If $\frac{7}{16}$ of a ship cost £51, what are $\frac{3}{32}$ of her worth? *Ans.* £10 18*s.* 6*d.* $3\frac{3}{7}q$.

4. At £$3\frac{5}{8}$ per *Cwt.* what will $9\frac{2}{3}$ *lb.* come to? *Ans.* 6*s.* $3\frac{5}{56}d$.

5. If $\frac{3}{4}$ *yd.* cost $\frac{5}{7}$ of a £. what will $\frac{9}{13}$ Ell English cost? *Ans.* 17*s.* 1*d.* $2\frac{3}{5}q$.

6. A man owning $\frac{3}{5}$ of a farm, sells $\frac{3}{4}$ of his share for £171; what is the whole farm valued at? *Ans.* £380.

Rule of Three Inverse in Vulgar Fractions.

EXAMPLES.

1. If $25\frac{1}{2}s$. will pay for the carriage of an Cwt. $145\frac{1}{4}$ miles, how far may $6\frac{1}{2}$ Cwt. be carried for the same money? *Ans.* $22\frac{9}{26}$ *miles.*

2. If the penny white-loaf weigh 7 *oz.* when a bushel of wheat cost 5*s.* 6*d.* what is the bushel worth when the penny white-loaf weighs $2\frac{1}{2}$ *oz.*? *Ans.* 15*s.* $4\frac{4}{5}d$.

3. How much shalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{3}{4}$ yards of cloth that is $1\frac{1}{4}$ yard wide? *Ans.* $11\frac{1}{4}$ *yards.*